ISSN 1512-1461

UDC 53.01 Theory and nature of phenomena

THE INVESTIGATION OF ASYMPTOTICAL BEHAVIOR OF THE AMPLITUDE FOR LARGE MOMENTA IN THE SO-CALLED "TWO-PARTICLE APPROXIMATION" AND NON-PHYSICAL LANDAU POLE PROBLEM

P.E. Agha-Kishieva[1,a)], S.G. Rahim-Zade[1,b)] and M.M. Mutallimov[2,c)]
[1)]Baku State University, Faculty of Physics,
[2)]Baku State University, Institute of Applied Mathematics
[a)]p_aghakishiyeva@outlook.com, [b)]sara.rehimzade@gmail.com, [c)]mutallim@mail.ru

Abstract

The oldest problem of quantum field theory - the asymptotic behavior at large momenta (or, at short-distances) we discussed and present our results of the investigation of asymptotical behavior of amplitude at short distances in four-dimensional scalar field theory. To formulate of our calculating model two-particle approximation of the mean-field expansion we have used an iteration scheme of solution of the Schwinger-Dyson equations with the fermion bilocal source. We have considered the nonlinear integral equations in deep-inelastic region of momenta. As result we have a non-trivial behavior of amplitude at large momenta: searched the location of non-physical poles (poles Landau) in amplitude at different coupling constants.

1. Introduction. As well known, Quantum Field Theory (QFT) –the theory of relativistic particle physics is the advanced version of the relativistic Quantum Mechanics[1]. QFT describes the properties and interactions of fundamental particles of matter, for example, electrons, photons, quarks and gluons, which are composed of other material objects. For example, a hydrogen atom is a bound state of an electron and proton interacting with an electromagnetic field (photons), and the proton, in turn, consists of quarks, interacting via gluons. The main characteristics include particle rest mass *m*, energy *E* and momentum *p*, which are interconnected by the known relation: $E^2 - p^2c^2 = m^2c^4$, which is satisfied in any inertial reference frame. Here *c* - light speed in the vacuum. In the rest system of the particle ($p = 0$) this ratio turns to the text color{magenta}{Einstein}'s famous formula: $E = mc^2$. If the relative momenta of the particles and their interaction energies are small compared to the rest mass, the motion of particles is described by quantum mechanics: each particle is mapped to the wave function ψ, which is the solution of the Schrödinger equation. Increasing the interaction energies, the usual quantum-mechanical description of particles becomes inapplicable, since there is a new physical phenomenon: creation and annihilation of particles. For example, during the scattering of high-energy photon ($\gamma -$quant) at the nuclei electrons and their antiparticles – positrons are produced. In turn, the electron and positron can annihilate, i.e. turn into photons. With further increase of the interaction energy more and more particles can be borne. The number of a new particles known today exceeds the hundreds. To describe the systems with a variable number of high-energy particles each class of fundamental particles is connected with quantized field, which consists the creation and annihilation operators of particles. Quantized field of the electron ψ is no longer the usual generalized function in quantum mechanics, and much more complex object - as operator's (operators-like generalized function). Such a quantized field describes, in general, all the particles of the class, i.e, electronic field describes all the electrons in the universe, the photon (electromagnetic field) - all photons, etc. The particles are divided into two categories - real particles existing in the initial and final stages of the physical process hysical and virtual, particles, which play a role only in the process of interaction between the particles. For real particles the usual relativistic relation between energy and

ISSN 1512-1461

momentum is valid. In high-energy physics so-called natural system of units is commonly used in which of light speed c and Planck's constant $\hbar$ equal to *one*: $c = \hbar = 1$. In this system of units using conventional 4-vector notation of relativistic mechanics $p = (p_0, \vec{p})$, $p_0 = E$, the ratio between the momentum and energy of real particles takes the simple form: $p^2 = p_0^2 - \vec{p}^2 = m^2$. As physicists say, the real particles are on the *mass shell*. For virtual particles, this relation is not satisfied: $p^2 \neq m^2$ i. e., *virtual particles are off the mass shell*. As in all physical experiments measured only the parameters of the initial and final states, the concept of virtual particles, of course, in no way does not violate the law of conservation of energy-momentum[1].

It is well known, that in QFT the basic mathematical objects of calculations are vacuum expectation values of products of fields $\langle 0|T\psi(x_1)\psi(x_2)...\psi(x_n)|0\rangle$. Here $|0\rangle$ - the vacuum state, i.e. state without real particles - *4-vector* $x = (x_0, \vec{x})$ coordinates in the usual *4-dimensional* space-time. Sign T indicates the chronological ordering of the field operators, i.e field operators are arranged in ascending order of time coordinates. Introduction of chronological ordering is necessary in order to take into account the *principle of causality*, i.e the correct sequence of events describing the particles interactions[1].

Knowing the vacuum expectation values, we can calculate all the physical characteristics of both the fundamental particles and composed of these objects, i.e., - *masses of the particles* and *bound states*, *scattering cross sections*, *lifetimes* of unstable particles, etc. Briefly theorists call the vacuum expectation values of products of fields *Green's functions*[1].

The simplest physically meaningful Green's function is the two-point Green's function, or propagator (particles propagation function): $D(x-y) = \langle 0|T\psi(x)\psi(y)|0\rangle$. Propagator depends only on a *4-dimensional variable* $x - y$. This fact is a reflection of the translational invariance of the theory, i.e independence of the physical phenomena of the coordinate system[1].

Simple propagators of the free fields are in momentum space is: $D_c(p) = \int dx e^{i(px)} D_c(x) \cong \frac{1}{m^2 - p^2}$. Note that in this formula p - is not a real particle momentum but the momentum variable Fourier conjugate to coordinate. As can be seen from this expression, the propagator has a pole singularity in the momentum variable. This fact is very general and is also valid for interacting fields. In other words, in QFT the pole of the Green's function implies the existence of a real particle with mass m. Massless particles (e.g, photons) correspond to the pole at the point $p^2 = 0$, and, accordingly, the propagator of a free photon has the form: $D_c(p) \cong \frac{1}{-p^2}$. The calculation of the Green's functions in the theory of interacting fields is a very difficult problem. For more than half a century, the exact physically meaningful solution of interacting quantum fields was not found. Therefore, the various approximate methods are of particular importance, among which the most important is the perturbation theory. Green's function of free fields are taken as the main approach. Interaction is considered as a small perturbation, which is physically quite reasonable for important case of quantum electrodynamics (QED) of interaction of electrons with photons, as the strength of interaction in this theory and is determined by the $\alpha = \frac{e^2}{4\pi} = \frac{1}{137} -$ a small and expansion parameter in the perturbation theory in QED[1].

By the mid-fifties of the last century, successful theoretical description of most of the well-known electrodynamic phenomena was given, including splitting of the electron levels in the hydrogen atom, the anomalous magnetic moment of the electron, etc. These successes have led theorists to investigate the limits of applicability of QED. In 1954-1955, Landau and his colleagues , published the results of their calculations, the asymptotic behavior of the Green functions of QED, i.e. behavior for large values of the momentum variable p^2[2]. These results were very strange, and

ISSN 1512-1461

further interpretation led them to a very sad for the QFT. It was found that when $\left|p^2\right| >> m^2$ asymptotic behavior of the photon propagator is described by the following formula: $D(p) \cong \frac{1}{-p^2}\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{-p^2}{m^2}\right)\right]^{-1}$, i.e., apart from the normal pole at $p^2 = 0$ the photon propagator has "ghost pole" at $p^2 = -m^2 e^{\frac{3\pi}{\alpha}}$. In accordance with the foregoing principles of QFT, such a pole corresponds to a particle with a 'negative squared mass'(?!)[2]. Such particles have never been observed experimentally, and their very existence contradicts the basic principles of particle physics. *Landau pole* cannot undo all the successes of QED, is very far from the energies attainable in experimental setups. Indeed, the value of 'Landau mass' according to the above formula is $M_L = 10^{28} m$, while the energy of the particles that can be achieved in the most modern plants do not exceed 10^7. Therefore, the effect of such a remote pole is negligible. But it exists, and it can not be ignored, especially since studies later confirmed the existence of such poles and in other models of QFT. There arose a *dual* and *a strange situation*. On the one hand, calculations based on perturbation theory described well the experimental data and the predictions of QED were always confirmed experimentally. On the other hand, QED was internally inconsistent, as contained in the statement of magnitude, the existence of which is contrary to the basic principles of the theory. This inner contradiction was inherent and other models of QFT, including models, claiming at the time to describe strong interactions[1,2].

Landau himself assessed the situation very pessimistic and made a very definitive conclusion: "Operators ψ containing unobservable information should disappear from the theory; and because the Hamiltonian can be built only from the operators, we need to come to the conclusion that the Hamiltonian method for strong interactions outlived its usefulness and should be buried, of course, with all the respect it deserves"[3]. In fact, Landau called completely abandons the concept of quantized fields in the describing of the interaction of high-energy particles. Instead, he proposed the creation of a new theory, which uses only the scattering amplitude and their analytic continuation. But the heroic efforts of many theorists to create this kind of theory, taken in the following years, unfortunately, yielded modest results. It turned out that the information contained in the field operators and compiled out of Lagrangians and Hamiltonians, replace virtually nothing. Remained the other way - to try to solve the problem within the framework of the QFT. But Vladimir Yakovlevich Fainberg from Lebedev Physical Institute (Moscow), listened to the personal word of Landau, "such non-physical poles must be reduced counter non-physical pole, in the summation of infinite number Feynman diagrams. Is reasonable: appropriate to look for a new nonperturbative approach!, i.e., a new method for summing Feynman diagrams, necessary!

A widespread opinion is formulated as a triviality of the quantum field models that is not asymptotically free in the sense of the improved coupling constant perturbative expansion. There is a rigorous theorem that the four-dimensional scalar field theory with φ^4 interaction on the lattice does not have an interacting continuum theory as its limit for zero lattice spacing, i.e. the theory is trivial. However, this argument is not fully conclusive due to an uncertainty of the continuous limit in this model. In our day the situation with triviality of φ^4 theory is vague as before, and recent papers in this topic maintain incompatible statements. So that in the models without asymptotic freedom the asymptotic short-distance region of strong coupling (exactly, concerning to weak coupling) is the difficultly at investigation, therefore a standard non-perturbative methods are too tethered to the weak-coupling region and not in full enough meaning to describes a short-distances for these models. Promising method for solving of problems for large momenta (or, short distances) demonstrated in works by Rochev [4]. It is new approximation in this directing and based on iteration scheme of solution of the Schwinger-Dyson equation (SDEs) with the fermion bilocal source. The present version of this method based on a system of SDEs for the single-particle and two-particle

ISSN 1512-1461

Greens functions. For standard QFT procedure, which is the beyond our knowledge, we will to investigate the following nonlinear second order Volterra-type integral equation for amplitude (for detail mathematical foundation, see [4]):

$$\frac{1}{y(t)} = \frac{1}{g} + l(t) + \int_0^t \overline{K}(t,\tau) y(\tau) d\tau \tag{1}$$

2. Numerical realization. For getting of the standard nonlinear second order Volterra-type integral equation in the form of Urysohn [5] make the following change,

$$\frac{1}{y(t)} = u(t)\,. \tag{2}$$

Then we get

$$u(t) = \int_0^t K(t,\tau,u(\tau)) d\tau + f(t) \tag{3}$$

on the segment $0 \le t \le T$, where $K(t,\tau,u(\tau)) = \dfrac{\overline{K}(t,\tau)}{u(\tau)}$ and $f(t) = \dfrac{1}{g} + l(t)$. Из (3) видно, что $u(0) = f(0)$. From (3) it is seen that $u(0) = f(0)$. Then for every $n > 1$ integer define a constant integration step $h = \dfrac{T}{n-1}$ and consider a discrete set $t_i = h(i-1)$, where $i = 1,2,...,n$. It's obvious that $t_1 = 0, t_n = T$. At the points of $t = t_i$,the equation (3) takes the form

$$u(t_i) = \int_0^{t_i} K(t_i,\tau,u(\tau)) d\tau + f(t_i)\,. \tag{4}$$

To obtain an explicit formula for the solution of the recurrence to find $u_i = u(t_i)$, the integral in the expression (4) using a quadrature formula of rectangles [6] on the segments $[t_i, t_{i+1}]$ with the selection of the value of the function at the left end $t = t_i$. Then we have

$$\int_0^{t_i} K(t_i,\tau,u(\tau)) d\tau \approx \sum_{j=2}^{i} A_{ij} K(t_i,\tau_j,u_{j-1}) \tag{5}$$

Label $f_i = f(t_i)$ and using (5) as in [5, 6] we obtain the relation of recursion formulas

$$\begin{aligned} u_1 &= f_1, \\ u_i &= \sum_{j=2}^{i} A_{ij} K(t_i,\tau_j,u_{j-1}) + f_i,\ i = 2,3,...,n \end{aligned} \tag{6}$$

Further, from (6) we obtain the solution of equation (1)

$$y_i = \frac{1}{u_i},\ i = 1,2,...,n$$

Example. Let some constant g, and the function $\overline{K}(t,\tau)$ and $l(t)$ are set as follows

$$\overline{K}(t,\tau) = \frac{\tau}{t} - 1 + \frac{1}{t}\log\frac{1+t}{1+\tau} + \tau\log\frac{t(1+\tau)}{\tau(1+t)}\,,$$

$$l(t) = (\frac{g}{2} - 1)\log(1+t) + (1-g)(1 - \frac{1}{t}\log(1+t)).$$

ISSN 1512-1461

By setting different values g, T and n we obtain approximate solutions whose graphs are shown below.

3.Conclution. By decreasing the values of g, Landau "point" m_L slowly increases (see Fig.1) and in terms of lower than g=0.99, the situation doubles (!) (see Fig.2) for Landau pole, which confirms the well known opinion: such non-physical poles must be reduced via counter non-physical pole, in the summation of infinite number Feynman diagrams. And less than g=0.1, a non-physical pole disappears (see Fig.3).

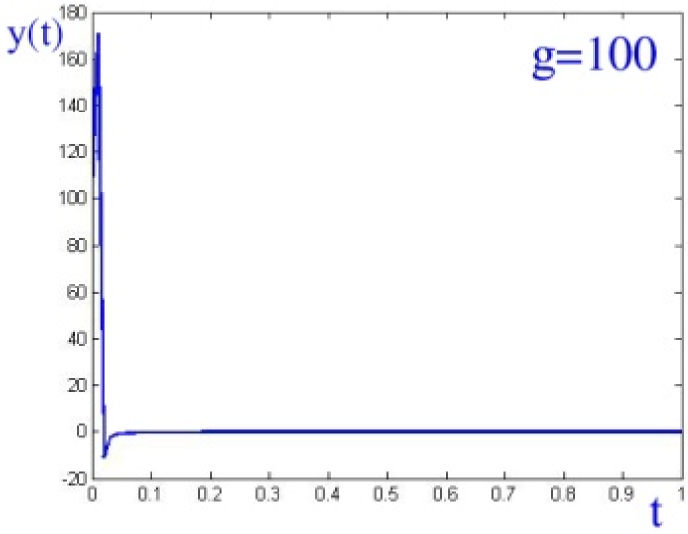

Fig.1.

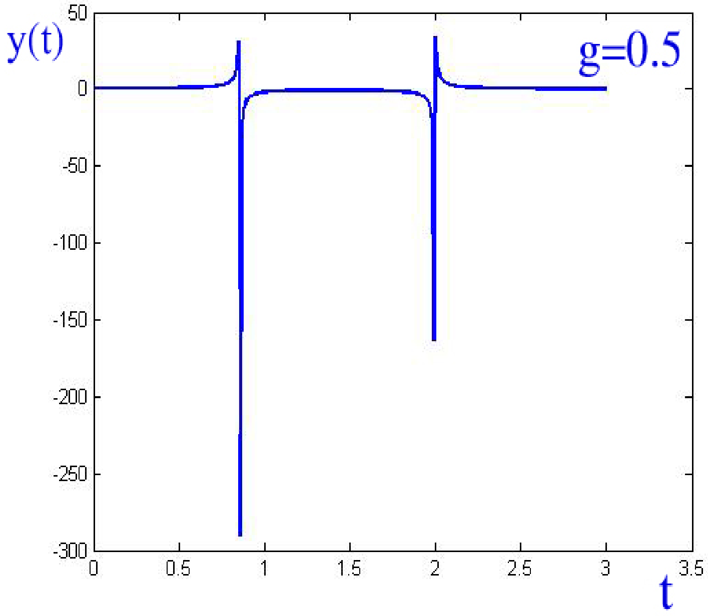

Fig.2.

ISSN 1512-1461

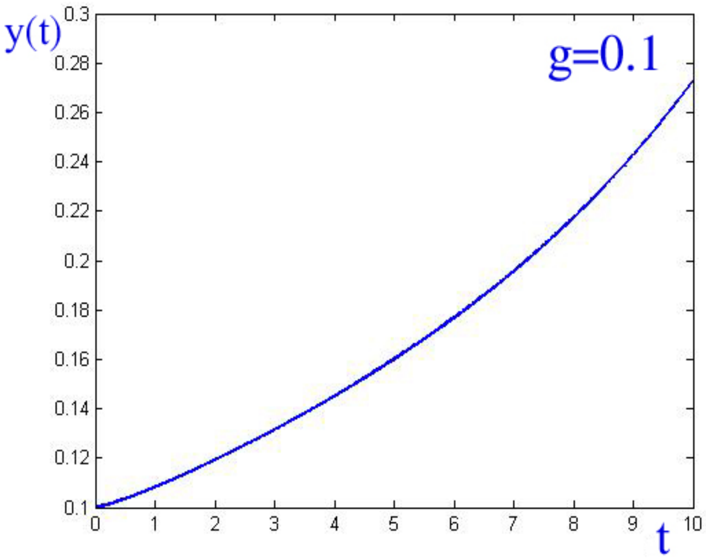

Fig.3.

REFERENCES

1. Cheng Ta-Pei and Li Lin-Fong. Gauge Theory of Elementary Particles Physics, Oxford, 1988; Peskin ME and Schroeder DV. An Introduction to Quantum Field Theory, Westview Press, 1995
2. Landau L D et al 1954 Dokl.Akad.Nauk Ser.Fiz. 95 1157; 1955 Dokl.Akad.Nauk Ser.Fiz. 102 489
3. *On the fundamental issues*, in Review: Theoretical Physics in the XX century, 1960
4. Rochev V E 2011 J.Phys.A:Math.Theor A44 305403;
5. *Манжиров А*.В., *Полянин А.Д. Методы решения интегральных уравнений*: *Справочник. М.*: *Факториал*, *1999*, *272*с.
6. *Калиткин Н.Н. Численные методы. М.*: *Наука*, *1978*, *512*с; Мосенцова Л.В. Численная реализация в системе Matlab квадратурных алгоритмов решения интегральных уравнений Вольтерра II рода в форме Гаммерштейна. Математичне та комп'ютерне моделювання. Серія: Технічні науки, 2010, Вип. 4, с. 156-161.

Article received: 2016-10-25