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ANALYSIS OF MAIN FACTORS IN LINEEAR ALGEBRA INSTRUCTIONS

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Abstract

This study investigates the main factors of instructional quality of linear algebra. Total 231 students in Hunan University of Technology completed the course requirements and the assigned questionnaires. Kaiser-Meyer-Olkin sample measure and Bartlett's sphere tests were employed to analyze the compatibility and reliability of the collected data. The factor analysis was subsequently implemented to find out the main factors in linear algebra instruction: instructional management of class, instructors' teaching level and students' interests to linear algebra. Several suggestions are provided at last for the improvement of instructional quality of linear algebra.

Keywords: *KMO sample; Bartlett's sphere tests; Factor analysis; Instructional quality; Linear algebra*

Introduction

In modern higher education, linear algebra is always one of the important basic courses in comprehensive university and regarded as the cornerstone of most subsequent courses. It has a lot of real-life applications in our modern multiple societies such as but not limited to genetics, networks, economics, coding theory, image compression, facial recognition and heat distribution (Toumasis, 2004). Learning linear algebra, covering facility of abstract definitions, construction of equations, approach to solving equations, analysis of linear spaces and interpretation of geometric figures, not only helps one to master the good mathematics literacy (for example the computational ability) but also provides a gateway for druging into higher mathematics (Trigueros, 2008). Consequently, a modern citizen trained by linear algebra should be able to interpret the comprehensive issue with understandable knowledge and express it in concise and accurate words. This definitely contributes to forming the good quality for people aiming at success, as in modern entrepreneurial era success required both good communication skills and efficient and concise expressions to convince others who might control the invested fund (Larson et,al, 2007; Trigueros, 2008).

Traditional instruction nowadays is faced with a strong technology shock and the instruction approach for some fundamental courses is facing new challenges and undergoing a deepening reform. On the other hand, cultivating students' logistic reasoning ability is becoming more urgent as the future demanding competency (Carlson et.al 1997; Sierpinska2000). Linear algebra, for new-coming students, plays a very important role in cultivating professionals with good mathematics literacy. Excellent linear algebra instructors should be able to reduce the new definitions and theories into the familiar knowledge mode and induce students to find similar ways to solve problems. Besides, they could enhance or reinforce the extraction of clues in geometric interpretations of linear algebra so that students are able to keep a relative long memory for future usage. (Carlson et.al 1997; Larson et,al, 2007; Toumasis, 2004; Trigueros, 2008) Therefore, instructional ways of current linear algebra teachers in higher education institutions deserve explored to promote the teaching level and instruction efficiency. In this study, we aim at giving a quantitative analysis of current situations of linear algebra instruction in China, which is based on questionnaires distributed to students from various majors.

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Some suggestions are provided according to derived findings.

Descriptions of the Investigation

Research subject

Total 248 students from Economic management, Electric Engineering, Mechanical Engineering and Civil Engineering of Hunan University of Technology took the linear algebra curriculum in the first semester of 2016 and 232 students completed the course requirements and were assigned to questionnaires. The basic information of investigated students was collected into the Table 1.

Table 1

Basic information of students

		Number	Ratio(%)
Gender	Male	131	56.4
	Female	101	43.6
Order of Desired Major at Admission	1	174	75.0
	2	37	15.9
	3	21	9.1

Note. In Order of Desired Major, "1" represents the most desired major; "2" stands for the secondary desired major; "3" is the undesired major.

Research design

To guarantee the rationale of the evaluation index, the questionnaire content was associated with the comprehensive understanding, interests and attitudes in learning, teaching contents and methods, which consisted of three parts. (each of them are five-point Likert-type responses).

- *About Teaching*

To investigate mathematics teachers' instructional attitudes and their approaches and levels, our questionnaire take two items from the Teaching and Learning Conceptions Questionnaire developed by Chan and Elliot (2004), which is consisted of traditional instruction dimension and constructivist instruction dimension.

1. Instructional management of class (excellent: 5, good: 4, general: 3, worse: 2, worst:1);
2. Instructional levels (highest: 5, high: 4, general:3, low:2, lowest: 1);
3. Instructional content to accept (easiest: 5, easy:4, general:3, hard:2, hardest:1)

- *About Learning*

To investigate students' perception of linear algebra learning, three items were selected from the Teaching and Learning Conceptions Questionnaire developed by Chan and Elliot (2004).

4. Interests to linear algebra (strongest:5, strong:4, general:3, weak:2, weakest:1);
5. Impact of linear algebra on other professional course (deepest:5, deep:4, general:3, shallow:2, none:1);
6. Independent completion of assignments (all:5, most:4, half:3, small part:2,unable:1);

- *About Students*

The Chinese version of the Beliefs about the Nature of Mathematics scale developed in the Teacher Education and Development Study in Mathematics (TEDS-M) was adopted in this study to assess future mathematics teachers' beliefs about the nature of mathematics, and satisfactory reliability has

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been indicated in various contexts (Tang & Hsieh, 2014).

7. Students source (single enrollment:1, full enrollment:0);
8. Gender (male: 1, female: 0);

For the convenience of analysis, the above eight items are denoted by X1, X2,…, X8 in the order of appearing.

Methods

The main object of this study is to make out which items in questionnaire will play more important roles in linear algebra instruction. So the factor analysis (De Vellis, 2012) is employed to explore the most potential factors that are not easy or unobservable. Factor analysis, if applicable, is definitely a powerful statistical means for data compression and improving data quality which can convert the original variables with mutual dependence and mutual influence into a few independent comprehensive factors.

We also conducted Kaiser-Meyer-Olkin (KMO) sample measure and Bartlett's sphere tests, prior to factor analysis (George & Mallery, 2003; Kline, 2000), to determine whether the data were suitable for factor analysis. Meanwhile, reliability analysis was also implemented to evaluate whether the questionnaire data is stable and reliable. It can make us know whether the questionnaire is scientific and effective. Besides, informal interviews and discussions from students and instructors were also included into the final survey.

Analysis of Results

Compatibility and reliability analysis

KMO sample measure is an efficient way to check the compatibility of factor analysis for the collected data. The closer to one KMO get, the more suitable for factor analysis implementation. Or in converse words, factor analysis becomes useless when KMO value is close to zero (George & Mallery, 2003; Kline, 2000). Bartlett's test, comparing the observed correlation matrix to the identity matrix, is a modification of the corresponding likelihood ratio test designed to make the approximation to the distribution better (Bartlett, 1937). In other words, it checks if there is a certain redundancy between the variables that we can summarize with a few number of factors. If the variables are perfectly correlated, only one factor is sufficient. If they are orthogonal, we need as many factors as variables. In this last case, the correlation matrix is the same as the identity matrix (George & Mallery, 2003; Kline, 2000).

Table 2
KMO and Bartlett's test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		0.740
Bartlett's Test of Sphericity	Approx. Chi-Square	64.135
	df	28
	P	0.000**

** stands for p<0.05, ** for p<0.01*

The derived KMO and Bartlett's test results were listed in Table 2, from which we can see that the KMO value 0.740 indicating an acceptable compatibility of factor analysis. Also the Bartlett's sphericity test 64.135, corresponding to the significance probability value (less than 0.01), indicated

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that there is a strong correlation between the variables, suitable for factor analysis.

In addition, Cronbach's Alpha test, also ranging from zero to one, was used to check the reliability of the questionnaire. In general, the reliability of the questionnaire is very low when Alpha value is less than 0.6 and becomes acceptable when Alpha value is greater than 0.6 and less than 0.8. Certainly, if Alpha value is greater than 0.8, the questionnaire has good reliability (De Vellis, 2012).

Table 3
Reliability Statistics

Cronbach's Alpha	N of items
0.752	8

From the obtained Cronbach's Alpha value shown in Table 3, the distributed questionnaire in our study seems to be reliable. So the factor analysis could be implemented.

Factor analysis

As the good compatibility and reliability of questionnaires and collected data, the factor analysis was subsequently applied and the obtained total variance interpretation quantity was tabulated into Table 4.

From Table 4, we can see that the variable correlation matrix has three largest eigenvalues which were greater than one (satisfying the principal component extraction principle) and explained 65.464% (cumulative contribution rate) information of the total variance. So the former three principal components were believed to contain most enough information of the original data. On the other side, to improve the efficient interpretation ability of synthetical factors, the rotated component matrix based on the maximum variance orthogonal rotation method was calculated and listed in Table 5.

Table 4
Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings	
Component	Total	% of Variance	Cumulative %	Total	% of Variance
1	2.681	29.384	29.384	2.681	29.384
2	1.818	19.925	49.309	1.818	49.309
3	1.474	16.155	65.464	1.474	65.464
4	.861	9.436	74.900		
5	.804	8.811	83.711		
6	.699	7.661	91.372		
7	.521	5.710	97.085		
8	.266	2.915	100.000		

Table 5
Rotated Component Matrix

Initial variable	Component		
	I	II	III
X1	.588	.833	.049
X2	.815	.373	.060
X3	-.008	.641	.007

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X4	-.031	-.302	.842
X5	.770	.009	-.017
X6	.614	-.039	-.075
X7	.266	.328	.458
X8	.018	.048	.124

It is obviously seen that the main factors that influence the instructional quality of linear algebra are: X2 (instructional level), X1(instructional management of class) and X4 (interests to linear algebra).

Findings and suggestions

The factor I is seen from Table 5 having a great load on the instructional level of the teachers, impact of linear algebra on other professional course and independent completion of assignments. Especially, the instructional level of teachers is the largest and this factor could be referred to the Instruction Level Factor (ILF). In addition, ILF occupies about 30% of the total variance and 44.8% of the extracted three main factors, which indicated the important role ILF plays in the linear algebra instruction. High-quality instruction in current ear requires teachers to adapt their instruction concepts and teaching methods to the gradual decline of students' quality and structured changes of students' resource. Meanwhile, new and technical teaching approaches should be explored to enlarge students' learning enthusiasm and cultivate students' initiative and consciousness, so that they are able to develop good study habits including previews before class, reviews after class and completion of assignments in time. Another practical way is to appoint a student for information communications who takes responsibility for keeping abreast of students' feedbacks and collecting difficult problems (including ones both from the class and homework) and hands them over the instructor. Then students' interests will be largely stimulated and their confidence will be greatly strengthened when they get satisfied solutions from teachers before learning new content.

Factor II attains the largest load on instructional management of class (83.3%) and could be called Instructional Management Factor (IMF). This factor accounts for 30.4% of the cumulative amounts of the three extracted factors and about 20% of the total variance interpretation. This implies that IMF holds a vital position in improving the instructional quality and plays an irreplaceable role in creating a positive and good learning environment. In addition, practical applications and experiences should be introduced into linear algebra curriculum, so that students' interests will be promoted and more attention will be paid on linear algebra learning.

Factor III has a relative large load on interests of linear algebra and instructional level. Especially, this factor accounts for 24.7% of the cumulative explanations for the three extracted factors and about 16% of the total variance interpretation. Therefore the proper linear algebra textbook is necessary for the instructional quality. As for the curriculum content, "necessary", "proper" and "application-oriented" principles should be adhered to fit the current basic university education. Under the premise of maintaining the logical structure of different chapters of linear algebra textbook, instructional content could be processed at various degrees according to professional training objectives. For example, students with preferences to liberal arts and management might focus on the calculation methods of linear equations. The organization of the whole instruction content and module should cater to their reception level with proper emphasis on the knowledge origin, the applicable conditions and visual images as much as possible, avoiding the strict derivation and proof of theorems. The language description of definitions and theroems in linear algebra should also be easy to be understood as far as possible. On the other hand, applications of linear algebra such as in image deblurring, google

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searching engine etc, might be introduced as most as possible to improve students' awareness of actively applying the learned knowledge to professional engineering practices (Lesh & Doerr, 2003; Lesh & English, 2005; Sriraman & Lesh, 2006). Besides, more praises and encouragements are necessary for students even for small progress. Also appreciations words but not the bruising words are more suggested in the classroom to make the students' attitude toward linear algebra range from passiveness to activeness. When the above aims are achieved, the linear algebra instruction is bound to be successful.

Conclusions and limitations

The current status quo of the decline of student quality brings a number of challenges to instructors in universities. It is particularly outstanding during the instructional process of the basic mathematical curriculum linear algebra. To improve the quality of instruction, the mainly determined factors are necessarily found out and efficient measures to be implemented. Data were collected by means of questionnaires and processed by factor analysis in this study. The obtained results show that the main factors that influence the teaching quality of linear algebra are instructors' teaching level, instructional management of class and students' interests to linear algebra. Due to the enrollment of students at different levels, some students have relatively poor mathematical foundation. They have no confidence in learning linear algebra and the learning consciousness is not strong. Therefore, instructors are suggested to not only impart knowledge technically, but also take the responsibility of cultivating students' quality of will, learning initiative and consciousness. In addition, teachers should also establish proper communication channels with students, integrate various information and communication technologies in teaching activities, redesign the new contents of curriculum reform, train students' ability of arithmetic and practical application and make students' linear algebra learning meet professional requirements in future.

There are some limitations in this study. (1) The factors in our questionnaire are not comprehensive enough; (2) The accuracy, effectiveness and reliability of the collected data is not high enough; (3) the surveyed objects are merely sampled from one university and might not be comprehensive.

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