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HESITANT TRIANGULAR FUZZY TOPSIS APPROACH FOR MULTIPLE ATTRIBUTES GROUP DECISION MAKING

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Abstract
In this article a decision support methodology for multiple attributes group decision making (MAGDM) problem is developed. The proposed methodology is based on the hesitant fuzzy TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method. The case is considered, when both the values and weights of the attributes are expressed in linguistic terms, given by all decision makers. Then, these linguistic terms are described by triangular fuzzy numbers. Following the TOPSIS method's algorithm, a relative closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to the fuzzy positive-ideal solution (FPIS), as well as to the fuzzy negative-ideal solution (FNIS). An example is shown to explain the procedure of the proposed methodology.

Keywords: *Multiple attributes group decision making, linguistic variable, hesitant triangular fuzzy set, TOPSIS method, ranking of alternatives.*

1. Introduction

The main objective of the multiple attributes group decision making (MAGDM) problem is to choose one best of the feasible alternatives or to rank all the alternatives that are evaluated by a group of decision-makers (DMs) based on multiple, often conflicting attributes. From this perspective, many practical applications (such as medical diagnostics, project management, personnel evaluation, business and finance management, supply chain management, etc.) represent a MAGDM problem.

To solve a MAGDM problem various methods have been proposed, among them one of the most popular is the TOPSIS approach. It was first developed by Hwang and Yoon [1]. In classical TOPSIS the weight and values of each attribute are presented by crisp numbers. However, the attribute and weight values cannot be expressed always with exact numbers due to uncertainty of decision makers' preferences, as well as the vagueness and complexity of attributes. Ignoring the fuzziness and uncertainty of evaluated objects leads to inadequate and non-acceptable decisions.

Processing fuzzy data in decision-making models is based on the concept of fuzzy sets introduced by Zadeh [2] and researched by Bellman and Zadeh [3]. As a generalization of a fuzzy set, Torra proposed notion of a hesitant fuzzy set and its application in decision-making [4, 5]. In this connection, many well-known MAGDM methods have been extended to take into account fuzzy types of values of attributes and their weights. The latter led to a great number of researches, in which evaluations of attributes involved in the decision making problems most often are expressed in fuzzy numbers, triangular fuzzy numbers, confidence intervals, hesitant fuzzy elements, interval-valued hesitant fuzzy elements and so on [6-14 and others].

However, a more natural representation of decision makers' assessments may be lingual expressions (linguistic variables) [7].

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In the proposed methodology both the values and weights of the attributes are expressed in linguistic terms, given by all decision makers. Then, these assessments are converted into the triangular fuzzy numbers. Decisions are made using TOPSIS method. Hence, proposed approach is based on hesitant triangular fuzzy TOPSIS decision making model.

The idea of the TOPSIS method consists in finding as optimal the alternative with the nearest distance from the fuzzy positive ideal solution (FPIS) and the farthest distance from the fuzzy negative ideal solution (FNIS). Following the TOPSIS method's algorithm, a relative closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to the FPIS, as well as to the FNIS.

The developed approach is applied to evaluation of investment projects with the aim of their ranking and identification of high-quality projects for investment. The article provides an investment decision-making example clearly illustrating the work of the proposed methodology.

2. Preliminaries

This Section presents some basic definitions and notations on the triangular fuzzy numbers, hesitant fuzzy sets and hesitant triangular fuzzy sets.

2.1. On the triangular fuzzy numbers

Definition 1 [2]. Let X be a reference set. A fuzzy set $\tilde{A}$ of X is defined by a membership function $\mu_{\tilde{A}}(x) \to [0,1]$, where $\mu_{\tilde{A}}(x)$, $\forall x \in X$, indicates the possible membership degree of x to $\tilde{A}$.

Definition 2 [15]. A triangular fuzzy number $\tilde{a}$ can be determined by a triplet (a_1, a_2, a_3). Its membership function $\mu_{\tilde{a}}(x)$ is defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & if \;\; x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1} & if \;\; a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2} & if \;\; a_2 \le x \le a_3, \\ 0 & otherwise. \end{cases}$$

Let $\tilde{a}$ and $\tilde{b}$ be two triangular fuzzy numbers (TFNs) given by the (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively. Some arithmetic operations are determined on these two numbers as follows:

$$\begin{aligned}
&1.\;\; \tilde{a}(+)\tilde{b} = (a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3); \\
&2.\;\; \tilde{a}(-)\tilde{b} = (a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1); \\
&3.\;\; \tilde{a}(\times)\tilde{b} = (a_1, a_2, a_3)(\times)(b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3),\; a_i > 0,\; b_i > 0; \\
&4.\;\; \tilde{a}(\div)\tilde{b} = (a_1, a_2, a_3)(\div)(b_1, b_2, b_3) = (a_1 / b_3, a_2 / b_2, a_3 / b_1),\; a_i > 0,\; b_i > 0; \\
&5.\;\; \lambda\tilde{a} = (\lambda a_1, \lambda a_2, \lambda a_3),\; \lambda > 0; \\
&6.\;\; (\tilde{a})^{-1} = (1/a_3, 1/a_2, 1/a_1),\; a_i > 0; \\
&7.\;\; \tilde{a} > \tilde{b} \;\; if \;\; a_2 > b_2; \text{ and } if \;\; a_2 = b_2 \;\; then \;\; \tilde{a} > \tilde{b} \;\; if \;\; a_1 + a_3 > b_1 + b_3; \; otherwise \; \tilde{a} = \tilde{b}.
\end{aligned} \tag{1}$$

Definition 3 [16]. The distance between two TFNs $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ is calculated by formula:

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$$d(\tilde{a},\tilde{b}) = \sqrt{\left((a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right)/3}\,. \qquad (2)$$

The following properties that will be used later are valid:

1. Two TFNs $\tilde{a}$ and $\tilde{b}$ are identical iff $d(\tilde{a},\tilde{b}) = 0$;
2. Let $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ be three TFNs. The TFN $\tilde{b}$ is closer to TFN $\tilde{a}$ that the other TFN $\tilde{c}$ iff $d(\tilde{a},\tilde{b}) < d(\tilde{a},\tilde{c})$.

A linguistic variable is a variable with values expressed in linguistic terms [17]. For instance, the linguistic variable “weight” may have the following values - very low, low, medium, high, very high and so on. These linguistic values can be represented by fuzzy numbers.

2.2. On the hesitant fuzzy sets and hesitant triangular fuzzy sets

Hesitant fuzzy set (HFS) was introduced by Torra and Narukawa in [4] and Torra in [5] as a generalization of a fuzzy set. In HFS the degree of membership of an element to a reference set is presented by several possible fuzzy values. This allows describing situations when DMs have hesitancy in providing their preferences over alternatives. The HFS is defined as follows:

Definition 4 [4, 5]. Let X be a reference set, a hesitant fuzzy set H on X is defined in terms of a function $h_H(x)$, which when applied to X returns a subset of [0,1]:

$$H = \{ < x, h_H(x) > | \, x \in X \}, \qquad (3)$$

where $h_H(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x \in X$ to H; $h_H(x)$ is called a hesitant fuzzy element (HFE).

In [18] Yu introduced the hesitant triangular fuzzy set (HTFS), where the membership degrees of
an element to a given set are expressed by TFN.

Definition 5 [18]. For a reference set X, a hesitant triangular fuzzy set T on X is defined in terms of a function $f_T(x)$ as follows:

$$T = \{ < x, f_T(x) > | \, x \in X \}, \qquad (4)$$

where $f_T(x)$ is a set of several triangular fuzzy numbers, representing the possible membership degrees of the element $x \in X$ to the HTFS T; $f_T(x)$ is called a hesitant triangular fuzzy element (HTFE).

3. MAGDM problem in hesitant fuzzy environment

Consider a multiple attributes problem for group decision making.

Assume that there are m decision making alternatives $-A = \{A_1, A_2, \ldots, A_m\}$, and the group $E = \{e_1, e_2, \ldots, e_\ell\}$ of t DMs (experts) evaluates them with respect to an n attributes $X = \{x_1, x_2, \ldots, x_n\}$.

Considering that the attributes have different importance degrees, the weighting vector of all attributes, given by the DMs, is defined by $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T$, where $\tilde{w}_j$ is the importance degree of jth attribute.

DMs provide evaluations of the attributes and of their weights in the form of lingual

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assessments – linguistic terms. Then, these assessments are expressed in triangular fuzzy numbers as shown in the following tables:

Table 1. Linguistic scale for the importance of the weight of each attribute

Linguistic term	Corresponding TFNs
Very low (VL)	(0, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.5, 0.7, 0.9)
Very high (VH)	(0.7, 0.9, 1.0)

Table 2. Linguistic terms for rating of alternatives

Linguistic term	Corresponding TFNs
Very poor (VP)	(0, 0, 3)
Poor (P)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Good (G)	(6, 8, 9)
Very good (VG)	(8, 10, 10)

Therefore, DMs joint assessments concerning each alternative represent HTFSs. A HTFS A_i of the ith alternative on X is given by $A_i = \{< x_j, f_{A_i}(x_j) > | x_j \in X\}$, where $f_{A_i}(x_j)$, $i = 1, 2, \ldots, m$;

$j = 1, 2, \ldots, n$ indicates the possible membership degrees of the ith alternative A_i under the jth attribute x_j, and it can be expressed as a HTFE $\tilde{t}_{ij}$.

Then a MAGDM problem in our case can be expressed in matrix format as follows

$$\tilde{T} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \cdots & \tilde{t}_{mn} \end{bmatrix} \end{array},$$

$$\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T,$$

$$E = \{e_1, e_2, \ldots, e_t\},$$

where $\tilde{T}$ is the fuzzy decision matrix (fuzzy hesitant triangular decision matrix), each element of which represents a HTFE $\tilde{t}_{ij}$.

4. The hesitant triangular fuzzy TOPSIS approach for MAGDM problem

The idea of TOPSIS method as applied to the problem of MAGDM is to choose an alternative with the nearest distance from the so-called fuzzy positive ideal solution (FPIS) and the farthest distance from the fuzzy negative ideal solution (FNIS).

The algorithm of practical solving an MAGDM problem can be formulated as follows:

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Step 1: Determine the attributes weighing vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T$ (see Table 1) using following rules.

If $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$ is the importance weight of the x_j attribute given by *k*th DM, then the aggregated fuzzy weight of each attribute are calculated as $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where

$$w_{j1} = \min_k \{w_{j1}^k\}, \qquad w_{j2} = \frac{1}{\ell} \sum_{k=1}^{\ell} w_{j2}^k, \qquad w_{i3} = \max_k \{ w_{j3}^k \}. \tag{5}$$

Step 2: Based on the DMs evaluations (see Table 2) construct the fuzzy decision matrix $\tilde{T} = (\tilde{t}_{ij})_{m\times n}$.

Step 3: Compute the score matrix - aggregate fuzzy decision matrix - as follows.

If $\tilde{t}_{ij}^{\,k} = (t_{ij1}^k, t_{ij2}^k, t_{ij3}^k)$ is an evaluation of the A_i alternative with respect to x_j attribute given by *k*th DM, then the aggregated fuzzy ratings $(\tilde{t}_{ij})$ of alternatives with respect to each attribute are given by $\tilde{t}_{ij} = (t_{ij1}, t_{ij2}, t_{ij3})$ where

$$t_{ij1} = \min_k \{ t_{ij1}^k \}, \qquad t_{ij2} = \frac{1}{\ell} \sum_{k=1}^{\ell} t_{ij2}^k, \qquad t_{ij3} = \max_k \{ t_{ij3}^k \}. \tag{6}$$

Step 4: Compute the normalized fuzzy decision matrix.

Because various attributes are usually measured in various units, it is necessary to transform various measurements of attributes into dimensionless attributes allowing to make comparisons between attributes.

The normalized fuzzy decision matrix $\tilde{R}$ is given by $\tilde{R} = [\tilde{r}_{ij}]_{m\times n},\ i = 1, 2, ..., m,\ j = 1, 2, ..., n$, where

$$\tilde{r}_{ij} = \left(\frac{r_{ij1}}{r_j^*}, \frac{r_{ij2}}{r_j^*}, \frac{r_{ij3}}{r_j^*} \right), \quad r_j^* = \max_i r_{ij3} \text{ for benefit (the more the better) attributes;} \tag{7}$$

$$\tilde{r}_{ij} = \left(\frac{r_j^-}{r_{ij3}}, \frac{r_j^-}{r_{ij2}}, \frac{r_j^-}{r_{ij1}} \right), \quad r_j^- = \min_i r_{ij1} \text{ for cost (the less the better) attributes.}$$

Step 5: Compute the weighted normalized fuzzy decision matrix.

The weighted normalized matrix $\tilde{V}$ is computed by multiplying the weights $(\tilde{w}_j)$ of evaluation attributes with the normalized fuzzy decision matrix $\tilde{R}$:

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n},\ i = 1, 2, ..., m,\ j = 1, 2, ..., n, \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij} (\times) \tilde{w}_j. \tag{8}$$

Step 6: Compute the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

Determine the FPIS A^+ and the FNIS A^- by formulas:

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, ..., \tilde{v}_n^+) = \left\{ \left\langle \tilde{v}_j^+ = \max_i \{ \tilde{v}_{ij} \} \right\rangle \mid j \in J'; \left\langle \tilde{v}_j^+ = \min_i \{ \tilde{v}_{ij} \} \right\rangle \mid j \in J'' \right\}, \tag{9}$$

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$$A^{-} = \left(\tilde{v}_1^{-}, \tilde{v}_2^{-}, ..., \tilde{v}_n^{-}\right) = \left\{\left\langle \tilde{v}_j^{-} = \min_i \left\{ \tilde{v}_{ij} \right\}\right\rangle \mid j \in J'; \left\langle \tilde{v}_j^{-} = \min_i \left\{ \tilde{v}_{ij} \right\}\right\rangle \mid j \in J'' \right\}, \quad (10)$$
$$i = 1,2,..,m, \; j = 1,2,..,n,$$

where J' is associated with a benefit attributes, and J'' - with a cost attributes.

Step 7: Compute the distance of each alternative from FPIS and FNIS.

The distances d_i^{+} and d_i^{-} of each weighted alternative A_i, $i = 1,2,..,m$ from the FPIS and the FNIS is computed as follows:

$$d_i^{+} = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^{+}); \quad d_i^{-} = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^{-}), \; i = 1,2,..,m, \quad (11)$$

where $d(.,.)$ is the distance between two fuzzy numbers (see formula (2)).

Step 8: Compute the relative closeness coefficient (RC_i) of each alternative to the FPIS A^{+}.

The relative closeness coefficient RC_i represents the distances to the fuzzy positive ideal solution A^{+} and the fuzzy negative ideal solution A^{-} simultaneously and is calculated as

$$RC_i = d_i^{-} \big/ (d_i^{+} + d_i^{-}), \; i = 1,2,..,m. \quad (12)$$

Step 9: Rank the alternatives.

Perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, m$ according to the relative closeness coefficients RC_i, in decreasing order by the rule: for two alternatives A_α and A_β $A_\alpha \succ A_\beta$, if $RC_\alpha > RC_\beta$, where $\succ$ is a preference relation on A. The best alternative will be the closest to the FPIS and farthest from FNIS.

5. Application to evaluation of investment projects

Consider an investment decision-making example adapted from [10] that clearly illustrates the work of the proposed methodology.

Suppose that in the competition for investment five construction companies are involved. The group of four DMs evaluates the investment projects taking into account the five attributes that are important for granting investment:

x_1 – business profitability;
x_2 – pledge guaranteeing repayment of the credit;
x_3 – location of construction object;
x_4 – workmanship;
x_5 – percent ratio of the pledge to the credit monetary amount.

In our concrete case, all attributes are of a benefit type. DMs give evaluations in form of linguistic terms. Assume the information of the importance weights of the attributes given by all DMs with linguistic assessments looks like:

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Table 3. The importance weights of the attributes in linguistic form

	Attributes				
	x_1	x_2	x_3	x_4	x_5
Weight	{H,VH,VH,H}	{L,M,L,H}	{M,H,VH,H}	{M,L,M,L}	{H,H,M,M}

Linguistic terms then will be transformed to the corresponding TFN shown in the Table 1. The HTFS obtained for the weight of each attribute are given below in Table 4.

Table 4. Weights of attributes in form of HTFS

	Attributes				
	x_1	x_2	x_3	x_4	x_5
Weight	{ (0.5,0.7,0.9), (0.7,0.9,1.0), (0.7,0.9,1.0), (0.5,0.7,0.9) }	{ (0.1,0.3,0.5), (0.3,0.5,0.7), (0.1,0.3,0.5), (0.5, 0.7, 0.9) }	{ (0.3, 0.5, 0.7), (0.5, 0.7, 0.9), (0.7, 0.9, 1.0), (0.5, 0.7, 0.9) }	{ (0.3, 0.5, 0.7), (0.1, 0.3, 0.5), (0.3, 0.5, 0.7), (0.1, 0.3, 0.5) }	{ (0.5, 0.7, 0.9), (0.5, 0.7, 0.9), (0.3, 0.5, 0.7), (0.3, 0.5, 0.7) }

Using the formula (5) we determine the attributes weighing vector $\tilde{w}$ as

$$\tilde{w} = \{(0.5,0.8,1.0),(0.1,0.45,0.9),(0.3,0.7,1.0),(0.1,0.4,0.7),(0.3,0.6,0.9)\}.$$

To evaluate the rating of alternatives with respect to each attribute DMs use the linguistic rating terms as given in Table 2. Aggregated results are presented in Table 5 as the linguistic fuzzy decision matrix.

Table 5. Ratings of alternatives given by DMs

		Attributes				
		x_1	x_2	x_3	x_4	x_5
Alternatives	A_1	{G,VG,VP,VG}	{G,G,F,VG}	{VG,VG,F,VG}	{VP,P,VP,VP}	{F,F,VP,VP}
	A_2	{P,P,VP,VP}	{P,F,VP,F}	{VP,P,VP,VP}	{P,VP,P,P}	{F,F,P,P}
	A_3	{F,VG,G,G}	{P,G,F,G}	{F,VG,VG,F}	{F,G,F,F}	{F,G,G,F}
	A_4	{G,VP,F,F}	{P,VP,G,P}	{G,G,G,G}	{VG,G,VG,VG}	{G, F,G, VG}
	A_5	{F,P,VP,VP}	{P,P,F,P}	{F,F,F,P}	{F,G,G,G}	{F,G,F,G}

These linguistic evaluations are transformed into HTF matrix by assigning the appropriate TFN as given in Table 2. Thus, we obtained the following hesitant triangular fuzzy decision matrix.

Table 6. The hesitant triangular fuzzy decision matrix $\tilde{T}$

		Attributes				
		x_1	x_2	x_3	x_4	x_5
Alternatives	A_1	{(6,8,9), (8,10,10), (0,0,3), (8,10,10)}	{(6,8,9), (6,8,9), (3,5,7),(8,10,10)}	{(8,10,10),(8,10,10), (3,5,7), (8,10,10)}	{(0,0,3), (1,3,5), (0,0,3), (0,0,3)}	{(3,5,7), (3,5,7), (0,0,3), (0,0,3)}
	A_2	{(1,3,5), (1,3,5), (0,0,3), (0,0,3)}	{(1,3,5), (3,5,7), (0,0,3), (3,5,7)}	{(0,0,3), (1,3,5), (0,0,3), (0,0,3)}	{(1,3,5), (0,0,3), (1,3,5), (1,3,5)}	{(3,5,7), (3,5,7), (1,3,5), (1,3,5)}
	A_3	{(3,5,7), (8,10,10), (6,8,9), (6,8,9)}	{(1,3,5), (6,8,9), (3,5,7), (6,8,9)}	{(3,5,7), (8,10,10), (8,10,10), (3,5,7)}	{(3,5,7), (6,8,9), (3,5,7), (3,5,7)}	{(3,5,7), (6,8,9), (6,8,9), (3,5,7)}
	A_4	{(6,8,9), (0,0,3), (3,5,7), (3,5,7)}	{(1,3,5), (0,0,3), (6,8,9), (1,3,5)}	{(6,8,9), (6,8,9), (6,8,9), (6,8,9)}	{(8,10,10), (6,8,9), (8,10,10),(8,10,10)}	{(6,8,9), (3,5,7), (6,8,9), (8,10,10)}

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A_5	{(3,5,7), (1,3,5), (0,0,3), (0,0,3)}	{(1,3,5), (1,3,5), (3,5,7), (1,3,5)}	{(3,5,7), (3,5,7), (3,5,7), (1,3,5)}	{(3,5,7), (6,8,9), (6,8,9), (6,8,9)}	{(3,5,7), (6,8,9), (3,5,7), (6,8,9)}

To avoid computational complexity in the decision making process the aggregate fuzzy decision matrix is developed using formula (6) from Step 3.

Table 7. The aggregate fuzzy decision matrix

		Attributes				
		x_1	x_2	x_3	x_4	x_5
Alternatives	A_1	(0, 7, 10)	(3, 7.75, 10)	(3, 8.75, 10)	(0, 0.75, 5)	(0, 2.5, 7)
	A_2	(0, 1.5, 5)	(0, 3.25, 7)	(0, 0.75, 5)	(0, 2.25, 5)	(1, 4, 7)
	A_3	(3, 7.75, 10)	(1, 6, 9)	(3, 7.5, 10)	(3, 5.75, 9)	(3, 6.5, 9)
	A_4	(0, 4.5, 9)	(0, 3.5, 9)	(6, 8, 9)	(6, 9.5, 10)	(3, 7.75, 10)
	A_5	(0, 2, 7)	(1, 3.5, 7)	(1, 4.5, 7)	(3, 7.25, 9)	(3, 6.5, 9)

Then, by formula (7) from Step 4, the ratings of each alternative attributes are normalized, and the result is listed in Table 8.

Table 8. The normalized fuzzy decision matrix $\tilde{R}$

		Attributes				
		x_1	x_2	x_3	x_4	x_5
Alternatives	A_1	(0, 0.7, 1.0)	(0.3, 0.775, 1.0)	(0.3, 0.875, 1.0)	(0, 0.075, 0.5)	(0, 0.25, 0.7)
	A_2	(0, 0.15, 0.5)	(0, 0.325, 0.7)	(0, 0.075, 0.5)	(0, 0.225, 0.5)	(0.1, 0.4, 0.7)
	A_3	(0.3, 0.775, 1.0)	(0.1, 0.6, 0.9)	(0.3, 0.75, 1.0)	(0.3, 0.575, 0.9)	(0.3, 0.65, 0.9)
	A_4	(0, 0.45, 0.9)	(0, 0.35, 0.9)	(0.6, 0.8, 0.9)	(0.6, 0.95, 1.0)	(0.3, 0.775, 1.0)
	A_5	(0, 0.2, 0.7)	(0.1, 0.35, 0.7)	(0.1, 0.45, 0.7)	(0.3, 0.725, 0.9)	(0.3, 0.65, 0.9)

Using formula (8) from Step 5 and item 3 of formula (1) the weighted normalized matrix $\tilde{V}$ is computed (see Table 9).

Table 9. The weighted normalized fuzzy decision matrix $\tilde{V}$

		Attributes				
		x_1	x_2	x_3	x_4	x_5
Alternatives	A_1	(0, 0.56, 1.0)	(0.03, 0.35, 0.9)	(0.09, 0.61, 1.0)	(0, 0.03, 0.35)	(0, 0.15, 0.63)
	A_2	(0, 0.12, 0.5)	(0, 0.15, 0.63)	(0, 0.053, 0.5)	(0, 0.09, 0.35)	(0.03, 0.24, 0.53)
	A_3	(0.15, 0.62, 1.0)	(0.01, 0.27, 0.81)	(0.09, 0.53, 1.0)	(0.03, 0.23, 0.63)	(0.09, 0.39, 0.81)
	A_4	(0, 0.36, 0.9)	(0, 0.16, 0.81)	(0.18, 0.56, 0.9)	(0.06, 0.38, 0.7)	(0.09, 0.47, 0.9)
	A_5	(0, 0.16, 0.7)	(0.01, 0.16, 0.63)	(0.03, 0.32, 0.7)	(0.03, 0.29, 0.63)	(0.09, 0.39, 0.81)

Following the fuzzy TOPSIS method's algorithm, we determine the FPIS A^+ and the FNIS A^- by formulas (9)-(10) and item 7 of formula (1), respectively:

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$$A^{+} = \{(0.15, 0.62, 1.0), (0.03, 0.35, 0.9), (0.09, 0.61, 1.0), (0.06, 0.38, 0.7), (0.09, 0.47, 0.9)\};$$
$$A^{-} = \{(0.0, 0.12, 0.5), (0.0, 0.15, 0.63), (0.0, 0.053, 0.5,), (0.0, 0.03, 0.35), (0.0, 0.15, 0.63)\}.$$

Then we calculate the distances d_i^+ and d_i^- of each alternative A_i from the FPIS A^+ and the FNIS A^- by formulas (11) from Step 7 and formula (2), respectively:

$$d_1^+ = 0.62627,\ d_2^+ = 1.5204,\ d_3^+ = 0.28567,\ d_4^+ = 0.38928,\ d_5^+ = 0.90227;$$
$$d_1^- = 1.01708,\ d_2^- = 0.08941,\ d_3^- = 1.20161,\ d_4^- = 1.29376,\ d_5^- = 0.72021.$$

Using formula (12) from Step 8 to calculate the relative closeness coefficient RC_i of each alternative A_i to the FPIS A^+ we obtain:

$$RC_1 = 0.61891,\ RC_2 = 0.05554,\ RC_3 = 0.80792,\ RC_4 = 0.76870,\ RC_5 = 0.44390.$$

Finally, we perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, 5$ according to the relative closeness coefficients RC_i, $i = 1, 2, \ldots, 5$ (see Step 9). Using the values of RC_i, the alternatives are ranked as:

$$A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_2.$$

From the obtained ranking of projects, it is possible to make a conclusion that the project A_3 will be the most preferable, while the project A_2 will be the least preferable choice of the decision. That means that when investing the capital only in one project, DMs prefer the investment project A_3, i.e. the project A_3 receives investment.

6. Conclusions

In the present work a novel approach for solving MAGDM problem is developed. The proposed methodology is the extension of TOPSIS method in the fuzzy environment using HTFS. Unlike other fuzzy TOPSIS methods, in this study HTFS are applied to processing the linguistic expressions used by decision makers.

If decision makers give linguistic evaluations of the importance of the attributes' weights and rating of the alternatives on the basis of these attributes, they only roughly describe these values because of the uncertainty and vagueness of the information and also due to hesitance of DMs preferences. In these cases, HTFS may be the best tool for solving decision-making problems with linguistic assessments. The use of HTFS will give an adequate conversion of linguistic terms used by the decision makers into an MAGDM problem.

The new aspects in the hesitant triangular TOPSIS approach have been used:

1. To identify both FPIS and FNIS the new formulas are applied, where the highest and lowest values of attributes are found by comparing the triangular numbers. These formulas also take into account both types of attributes - a benefit attributes, as well a cost attributes;
2. The developed approach was applied in the problem of investment decision making.

Based on proposed approach the software package has been developed and used to rank investment projects in the real investment decision making problem. The application and testing of the software was carried out based on the data provided by the “Bank of Georgia”. The results are illustrated in the example.

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