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Mathematical Model of Transformation of Two-party Elections to Three-party Elections

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Abstract

In work the new nonlinear mathematical model of transformation of two-party elections to three-party elections is offered. In model it is supposed that two parties participated in the previous elections and as a result of elections one of parties became pro-governmental and the second oppositional. Before the following elections to political arena there is the second opposition party and on the subsequent elections two oppositional and one pro-governmental parties already participate.

In that special case, when demographic factor of elections for all three parties is equal to zero, coefficients of involvement of voters are constant, but are excellent during the periods of two-party membership and three-party membership, Cauchy's task in the first interval for system of two, and in the second interval of three nonlinear differential equations is solved analytically exactly. Taking into account indicators of appearances at the next elections of all three parties supporting voters, certain falsifications of voices of opposition parties, necessary conditions for a victory of pro-government party are found. The found exact analytical solutions allow all three parties to select strategy for a win of elections.

Keywords*: two-party, three-party, elections, analytical solutions.*

Introduction

Mathematical and computer modeling has been widely recognized in such disciplines as sociology, history, political science, and others [1, 2]. There is an interest in creation of a mathematical model, which would give the opportunity to determine the dynamics of changes in the number of voters of political subjects during the election period. Elections can be divided into two parts: the two-party and multi-party elections.

In [3-5] quantities of information streams by means of new mathematical models of information warfare are studied. By information warfare the authors mean an antagonism by means of mass media (an electronic and printing press, the Internet) between the two states or the two associations of states, or the economic structures (consortiums) conducting purposeful misinformation, propagation against each other.It was shown that in case of high aggression of the contradictory countries, not preventive image the operating peacekeeping organizations won't be able to extinguish the expanding information warfare.

In works [6,7] linear and nonlinear mathematical models of information warfare, and also optimizing problems are considered.

In [8] the nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutes is offered. The model is described by Cauchy's problem for nonlinear non-homogeneous system of the differential equations. Confronting sides in extend of provocative statements, the third side (the peacekeeping international organizations) extends of soothing statements, interstate authoritative institutes the peacekeeping statements call the sides for the termination of information warfare. In that specific case, modes of information warfare "aggressor- victim", for the third peacekeeping side are received exact analytical solutions, and functions defining number of the provocative statements distributed by the antagonistic sides satisfy to Cauchy's problems for Riccati certain equations which are solved by a numerical method. For the general model computer modeling is carried out and shown that irrespective of high aggression of confronting sides, interstate authoritative institutes will be able to extinguish information warfare and when for this purpose efforts of only the international organizations insufficiently.

The article [9, 10] concerns of Chilker task is entered - refers to the boundary value problem for a system of ordinary differential equations and optimal control problem. In Chilker tasks right boundary conditions are set in different, uncommitted time points for different coordinates of the unknown vector — functions. Proposed methods solutions of Chilker tasks.

In works [11, 12] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

These papers [13-17] present the nonlinear mathematical model of the public or the administrative management (or the macro and micro model). The cases of both constant and variable pressure forces on freethinking people were analyzed. Exact analytical decisions which determine dynamics of a spirit both free-thinking people, and operated (conformists) of people by time are received. During this analyses various governance systems were considered: a liberal, democratic, semi dictatorial and dictatorial.

These works [18-24] considered a two or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections when coefficients are constant. The assumption was made that the number of voters remain the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received. The conditions under which opposition party can win the upcoming elections were established.

These works [25-28] considered a two-party (pro-government and opposition parties) nonlinear mathematical model of elections with variable coefficients.

In work [29] proposed the nonlinear mathematical model with variable coefficients in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the pro-government and two opposition parties from election to election. The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In the particular case obtained exact analytical solutions. The conditions have been identified under which the opposition can win the forthcoming elections, and in some cases, the pro-government party can stay in power. In general Cauchy problem was solved numerically using the MATLAB software package.

In [30] consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation). Also it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states.

In [31] mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful

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states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilating side. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found. The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

In [32] mathematical modeling of nonlinear process of the assimilation taking into account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered. In model three objects are considered: the population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; the population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; population of the third state formation which is exposed to bilateral assimilation from two powerful states or the coalitions.

For nonlinear system of three differential equations of the first order are received the two first integral. Special cases of two powerful states assimilating the population of small state formation (autonomy), with different initial number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in all cases there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found.

In [33] work mathematical modeling of nonlinear process of two-level assimilation taking into account demographic factors of three sides is offered.

In [34] is proposed the nonlinear mathematical model with variable coefficients in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the pro-government and two opposition parties from election to election. The model considers four objects: state and administrative structures, acting by means of administrative resources for opposition-minded voters with the aim to win their support for the pro-government party; voters who support the first opposition party; voters who support the second opposition party; voters who support the pro-government party. The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In particular, we assume that in the period between elections coefficients of "attracting" voters are exponentially increasing function of time. In the particular case we obtain exact analytical solutions. The conditions have been identified under which the opposition can win the forthcoming elections, and in some cases, the pro-government party can stay in power.In general Cauchy problem was solved numerically using the MATLAB software package.

In [35] the nonlinear continuous mathematical model of interference of fundamental and applied researches on the example of one, perhaps closed for external customers, of scientifically – research institute (micro-model) is considered. For a special case, Cauchy's problem for nonlinear system of differential equations of first order is definitely decided analytically. In more general case

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based on Bendikson's criteria the theorem of not existence in the first quarter of the phase plane of solutions of closed integral curves is proved. Conditions on model parameters in case of which existence of limited solutions of system of nonlinear differential equations is possible are found.

1. General mathematical model of transformation of two-party elections to three-party elections

We will consider the general mathematical model of transformation of two-party elections to three-party elections, from the accounting of demographic factor of elections which we have an appearance:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t)-\alpha_2(t))N_1(t)N_2(t) + (\alpha_1(t)-\alpha_3(t))N_1(t)N_3(t) - F_1(N_1(t),t) + \gamma_1(t)N_1(t) \\ \dfrac{dN_2(t)}{dt} = (\alpha_2(t)-\alpha_1(t))N_1(t)N_2(t) + (\alpha_2(t)-\alpha_3(t))N_2(t)N_3(t) - F_2(N_2(t),t) + \gamma_2(t)N_2(t) \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t)-\alpha_1(t))N_1(t)N_3(t) + (\alpha_3(t)-\alpha_2(t))N_2(t)N_3(t) + F_1(N_1,t) + F_2(N_2,t) + \gamma_3(t)N_3(t) \end{cases} \quad (1.1)$$

$t \in (0,T]$,

$$N_1(0) = N_{10}, \quad N_2(0) = 0, \ N_3(0) = N_{30}, \quad N_{30} > N_{10} \quad (1.2)$$

$$N_2(t) \equiv 0, \quad F_2(N_2(t),t) \equiv 0, \quad t \in [0,T_1), \ T_1 < T$$

$$N_2(T_1) = N_{20} > 0$$

In this nonlinear mathematical model (1.1), (1.2) all coefficients are variables and demographic factors are taken into consideration.

Equations (1.1) is defined in the interval $t \in (0,T]$, and initial conditions (of Cauchy) $t = 0$ moment of time.

We look for the solution of the Cauchy problem on the segment $t \in [0,T]$ in the class of continuous differentiable functions

$$N_1(t), N_2(t), N_3[t] \in C^1[0,T]. \quad (1.3)$$

In a nonlinear system of differential equations (1.1):

$N_1(t), N_2(t), N_3(t) -$ are the numbers of supports of two opposition and one ruling party ($N_3(t)$) at time *t*,

$t = 0$ - is the time of previous elections, when one of the parties $N_3(t)$ won the elections and became the ruling party;

$t = T -$ is the time of the following elections (in many cases *T*= 4 years or 1460 days);

$t = T_1$($0 < T_1 < T$) - time-point, when to political arena there is the second opposition party $N_2(t)$;

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$\alpha_1(t), \alpha_2(t), \alpha_3(t)$ -- are the coefficients of attracting votes by the first and second opposition party and the ruling party at time *t*. They largely depend on the action programs, as well as financial, technological and informational capacities of the political parties.

$F_1(N_1(t),t), F_2(N_2(t),t)$ -- are the continuous positive functions, that define the scale of used administrative resources.

$\gamma_1(t), \gamma_2(t), \gamma_3(t)$ -- are the coefficients that describe demographic changes of the parties.

2. A special case with zero demographic factor of elections

We will consider a special case when the functions characterizing use of administrative resources are linear concerning the first variables, demographic factors of elections of all three parties are equal to zero and parameters of model are constant.

$$F_1(N_1(t),t) = \beta_1 N_1(t), \quad F_2(N_1(t),t) = \beta_1 N_2(t), \qquad (2.1)$$
$$\gamma_1(t) = \gamma_2(t) = \gamma_3(t) \equiv 0, \ t \in [0,T],$$

$$\alpha_1(t) = \alpha_1 = const > 0, \alpha_2(t) = \alpha_2 = const > 0, \alpha_3(t) = \alpha_3 = const > 0$$
$$\beta_1 = const > 0, \ \ \beta_2 = const > 0.$$

Taking into account (2.1), (1.1) and (1.2) will correspond in the following look

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1 - \alpha_3)N_1(t)N_3(t) - \beta_1 N_1(t) \\ \dfrac{dN_3(t)}{dt} = (\alpha_3 - \alpha_1)N_1(t)N_3(t) + \beta_1 N_1(t) \end{cases} \quad t \in (0,T_1] \ , \qquad (2.2)$$
$$N_1(0) = N_{10}, \quad N_3(0) = N_{30}, \quad N_{30} > N_{10}$$

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_{11} - \alpha_{31})N_1(t)N_3(t) + (\alpha_{11} - \alpha_2)N_1(t)N_2(t) - \beta_{11}N_1(t) \\ \dfrac{dN_2(t)}{dt} = (-\alpha_{11} + \alpha_2)N_1(t)N_2(t) + (\alpha_2 - \alpha_{31})N_3(t)N_2(t) - \beta_2 N_2(t) \\ \dfrac{dN_3(t)}{dt} = (-\alpha_{11} + \alpha_{31})N_1(t)N_3(t) + (-\alpha_2 + \alpha_{31})N_3(t)N_2(t) + \beta_{11}N_1(t) + \beta_2 N_2(t) \end{cases} \qquad (2.3)$$
$$t \in (T_1,T], \quad N_1(T_1) = N_{11}, \qquad N_2(T_1) = N_{20} > 0, \quad N_3(T_1) = N_{31}$$

Thus, we get the two Cauchy problem (2.2), (2.3).

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The solution of the Cauchy problem (2.2) has the form

$$N_1(t)+N_2(t)=N_{10}+N_{30}=a, \quad t\in[0,T_1]$$

When $b=a(\alpha_1-\alpha_3)-\beta_1\neq 0$,

$$\begin{cases} N_1(t)=\dfrac{N_{10}(1+\alpha_1-\alpha_3)\exp(bt)}{1+(\alpha_1-\alpha_3)\exp(bt)} \\ N_3(t)=\dfrac{a+[N_{30}(\alpha_1-\alpha_3)-N_{10}]\exp(bt)}{1+(\alpha_1-\alpha_3)\exp(bt)} \end{cases} \tag{2.4}$$

When $b=a(\alpha_1-\alpha_3)-\beta_1\neq 0$,

$$\begin{cases} N_1(t)=\dfrac{N_{10}}{1+N_{10}(\alpha_1-\alpha_3)t} \\ N_3(t)=\dfrac{N_{30}+a(\alpha_1-\alpha_3)N_{10}t}{1+(\alpha_1-\alpha_3)N_{10}t} \end{cases} \tag{2.5}$$

Therefore, it agrees (2.4), (2.5), we will receive initial values for Cauchy's task (2.3)

$$N_1(T_1)=N_{11}=\begin{cases} \dfrac{N_{10}(1+\alpha_1-\alpha_3)\exp(bT_1)}{1+(\alpha_1-\alpha_3)\exp(bT_1)}, a(\alpha_1-\alpha_3)\neq\beta_1 \\ \dfrac{N_{10}}{1+N_{10}(\alpha_1-\alpha_3)T_1}, a(\alpha_1-\alpha_3)=\beta_1 \end{cases} \tag{2.6}$$

$$N_3(T_1)=N_{31}=\begin{cases} \dfrac{a+[N_{30}(\alpha_1-\alpha_3)-N_{10}]\exp(bT_1)}{1+(\alpha_1-\alpha_3)\exp(bT_1)}, a(\alpha_1-\alpha_3)\neq\beta_1 \\ \dfrac{N_{30}+a(\alpha_1-\alpha_3)N_{10}T_1}{1+N_{10}(\alpha_1-\alpha_3)T_1}, a(\alpha_1-\alpha_3)=\beta_1 \end{cases} \tag{2.7}$$

From (2.3) it is easy to receive the first integral of system

$$N_1(t)+N_2(t)+N_3(t)=N_{11}+N_{20}+N_{31}=c, \quad t\in[T_1,T] \tag{2.8}$$

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If takes place $c(\alpha_{11}-\alpha_2)=\beta_{11}-\beta_2$, then

$$N_1(t)=\frac{N_{11}}{N_{20}}N_2(t), \quad t\in[T_1,T], \; p=\frac{N_{20}}{N_{11}}.$$

Then the exact solution of the Cauchy problem (2.3) has the form

$$\begin{cases} N_1(t)=\dfrac{\dfrac{(\alpha_{11}-\alpha_{31})c-\beta_{11}}{\alpha_{11}-\alpha_{31}+p(\alpha_2-\alpha_{31})}N_{11}\exp\{[(\alpha_{11}-\alpha_{31})c-\beta_{11}](t-T_1)\}}{\dfrac{(\alpha_{11}-\alpha_{31})c-\beta_{11}}{\alpha_{11}-\alpha_{31}+p(\alpha_2-\alpha_{31})}+N_{11}\exp\{[(\alpha_{11}-\alpha_{31})c-\beta_{11}](t-T_1)\}-N_{11}} \\ N_2(t)=pN_1(t) \\ N_3(t)=c-(p+1)N_1(t) \end{cases} \quad t\in[T_1,T] \qquad (2.9)$$

Analysis exact solutions of (2.9) shows that under full turnout of voters supporting the respective parties, the pro-government party wins the next elections, if the inequality

$$N_3(T)>N_1(T)+N_2(T). \qquad (2.10)$$

If the relative appearances $0<k_1<1$, $0<k_2<1$, $0<k_3<1$ of voters supporting the respective parties, and consideration relative to the falsification of the votes of the opposition parties $0<\delta_1<1$, $0<\delta_2<1$, you can specify the inequality under which the pro-government party wins the next elections

$$k_3N_3(T)>k_1(1-\delta_1)N_1(T)+k_2(1-\delta_2)N_2(T). \qquad (2.11)$$

It should also be noted that the implementation in (2.10), (2.11), the opposite inequality, pro-government party will lose the next elections, and the equality in relations (2.10), (2.11) the number of votes the pro-government and two opposition parties together will be the same (equal to the number of seats in Parliament for the pro-government party and opposition parties, which could lead to a political crisis associated with the inability of acquisition of the government or to the coalition government).

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