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DEPENDENCES OF AMOUNT AND DURABILITY OF THE ACQUIRED KNOWLEDGE FROM DURATION OF LESSONS: RESULTS OF IMITATING MODELING

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Abstract:
The computer model of mastering of the logically connected material is considered, the dependence of knowledge quantity and forgetting time of half of the acquired information from duration of lesson is investigated. The program on Pascal and turned out graphs are presented.

Key words: didactic system, computer modeling, training, teacher, learner.

Introduction

More than half-century back on crossing of didactics and mathematics the mathematical theory of training (MTT) was emerged; it was engaged in research of process of training by a method of mathematical modeling. The development of information technologies led to appearing of the method of imitating (computer) modeling that essentially has changed MTT, simplified and speed up the solving of complex tasks, raised clarity of turning out results. Computer modeling of system "teacher – learner" supposes a creation of the computer program simulating its behavior, and realization with it of a series of calculating experiments under various conditions with the purpose of an establishment of its functioning regularities and estimation of efficiency of various management strategy.

In the article the advanced model of learning and forgetting of the logically connected material [1 - 3], which takes into account decrease of the learner's working capacity with flow of time, is offered, and with its help the dependence of durability (strength) of knowledge mastering from duration of training is studied. At this it uses the system approach [4], methodology of soft systems [5], methods of the mathematical and computer modeling of behavior and training [6, 7], works on cognition psychology (understanding, assimilation, forgetting) [8].

The used approach assumes that the educational material is considered as a set of N separate ideas or information blocks [2]. Each block consists of M learning material elements (LME-s), ordered and connected with logic links. To understand any new idea the pupil should solve the given intellectual problem (i.e. to study a sequence of all LME-s), included into the structure of the given information block, in first time. When the pupil has acquired all LME-s of the given idea and, solving the educational task, again goes through their sequence, in second (fifth or tenth) time, he comes back to the concrete cognitive situation. This happens without active involvement of thinking and is called understanding-recollection. The offered model also takes into account the reduction of the pupil's working capacity.

The main part

Knowledge of the given $(i, j)-$LME is defined by probability $p_{i,j}$ of the correct answer to the corresponding elementary question (or solving of the simple task). For the simplicity it is possible to imagine a two-dimensional array of N lines and the M columns in which each element is equal to probability $p_{i,j}$ of the corresponding LME remembering. All LME-s can be divided into two categories: 1) LME-s well-known to the pupil; the probability of the correct answer for them is $p_{i,j} = 1$; 2) LME-s poorly-known to the pupil before the beginning of training, $p_{i,j} =$ 0 – 0,1; their quantity is equal n. In process of training the probabilities $p_{i,j}$ of new LME-s grows, and af-

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ter end of training – decreases because of forgetting. The probability of the given $i-$th idea reproduction by the pupil is equal to the product of the probabilities $p_{i,j}$ for all poorly-known LME-s making this idea: $P_i = p_{i,1}p_{i,2}\cdots p_{i,n}$.

The studied material can be specified by the following characteristics: 1) the number N of blocks (chains of LME-s); 2) the length of the blocks M, which is equal to quantity of elements, included in it; 3) the share D $(0 \le D \le 1)$ of LME-s known to the pupil a priori, that is before training; 4) the average coefficient of assimilation a for each LME. When studying the $j-$th LME from the $i-$th block during the time Δt, the probability of the student's correct answer to the corresponding elementary question grows according to the law: $p_{i,j}^{k+1} = p_{i,j}^{k} + aW(1-p_{i,j}^{k})\Delta t$; the counter of addresses $s_{i,j}$ for the given $(i, j)-$LME increases by 1. As the number of uses $s_{i,j}$ for this LME increases, the forgetting coefficient decreases according to the law: $g_{i,j} = 10^{-6} + 0{,}002\exp(-5\cdot 10^{-4}s_{i,j})$. If the pupil does not work with $(i, j)-$LME then, due to forgetting, knowledge of this LME during the time Δt decreases so: $p_{i,j}^{k+1} = p_{i,j}^{k}(1-g_{i,j}\Delta t)$. In order to take into account that before training some LME-s are known for pupil, a matrix $d_{i,j}$ is created, whose elements with a given probability D are equal to 1, and with probability $(1-D)$ – 0. The sums of probabilities $p_{i,j}^{k}$ for all LME-s with $d_{i,j} = 0$ and $d_{i,j} = 1$ are denoted by z_1 and $z_2 \approx D\cdot N\cdot M$ accordingly. At training and forgetting changes only z_1. To take into account reduction of the pupil's working capacity, we admit that coefficient of assimilation is connected with the duration of training t by logistic law:

$$W(t) = 0{,}01 + \frac{0{,}19}{1+\exp(2{,}5\cdot 10^{-3}(t-\theta))},$$

where $\theta = 5000$ is the time from a beginning of a lesson, at which second summand decreases in 2 times. If the number of addresses $s_{i,j}$ to given LME, the coefficient of mastering a and learner's working capacity W are growing, the time of solving t_S of the corresponding task decreases, aspiring to some limit: $t_S = dt(0{,}1 + 0{,}9\exp(-0{,}001\cdot s_{i,j}))/(aW)$.

Program PR–1. Computer model of mastering and forgetting of information blocks.

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{$N+}uses crt, graph; const dt=0.01; Y=650; N=30; M1=25; a=4; b=2.5E-3; Mt=0.025; Mz=0.16;
Mp=1; var t0,dtt,aw,t,tr,g,S0,W,SR,SP,SQ,P0: single; i,j,k,k1,kk,bb,c,fl,AA,M,DV,MV,DU:
integer; p: array [1..N,1..M1] of single; dd: array [1..N,1..M1] of integer; s: array [0..N,
0..M1]of single;
Procedure Test;
begin SR:=0; For k:=1 to 100 do  begin For i:=1 to N do  begin P0:=1; For j:=1 to M do P0:=P0
*p[i,j]; fl:=0; k1:=0; Repeat inc(k1); If P0>random(100)/100 then fl:=1; until (k1=3)or(fl=1);
If fl=1 then SR:=SR+1; end;  end; circle(10+round(t*Mt),Y-round(Mz*SR),2); SP:=0; For i:=1 to
N do For j:=1 to M do  begin  If dd[i,j]=1 then SP:=SP+p[i,j]; end;  circle(10+round(t*Mt),Y-
round(SP*Mp),1);SQ:=0; For i:=1 to N do For j:=1 to M do begin If dd[i,j]=0 then SQ:=SQ+p[i,j];
end; circle(10+round(t*Mt),Y-round((SQ+SP)*Mp),1); (*S0:=0; For i:=1 to N do For j:=1 to M do
S0:=S0+s[i,j];circle(10+round(t*Mt),Y-round(0.05*S0/l[i]/str),2);*) end;
Procedure Zabivanie;
begin For i:=1 to N do For j:=1 to M do If dd[i,j]=0 then begin g:=1E-6+0.002*exp(-5E-4*s[i,j]
); p[i,j]:=p[i,j]*(1-g*dtt); end; end;
Procedure Obuchen;
begin For i:=1 to N do For j:=1 to M do begin If dd[i,j]=0 then begin s[i,j]:=s[i,j]+1; p[i,j]
:=p[i,j]+a*W*(1-p[i,j])*dt; tr:=dt*(0.1+0.9*exp(-1E-3*s[i,j]))/a/W; t:=t+tr; circle(10+round(t
*Mt),Y-round(15000*tr),1); end; end; end;
BEGIN DV:=Detect; InitGraph(DV,MV,''); Randomize; DU:=8000; line(0,Y,1600,Y); line(10,0,10,Y);
line(round(10+Mt*DU),0,round(10+Mt*DU),800);  circle(10+round(1E+4*Mt),Y+10,5);  For i:=1 to N
do For j:=1 to M1 do begin If random(100)/100<0.3 then dd[i,j]:=1 else dd[i,j]:=0;If dd[i,j]=1
then p[i,j]:=1; end; Repeat circle(10+round(t*Mt),Y-round(W*1000),1); AA:=300;bb:=5; t0:=t; If
t<DU then begin W:=0.01+0.19/(1+exp(b*(t-6000))); M:=10; Obuchen; AA:=100; bb:=1; end else t:=
t+20*dt; dtt:=t-t0; Zabivanie; inc(c); If c>AA then begin c:=0; Test; end;
until (KeyPressed)or(t>5E+5); CloseGraph; END.
```

The pupil's condition in each moment of time is determined by a matrix of probabilities 1, 2, …, $p_{i,j}$, where $j = 1, 2, \ldots, M$. For an estimation of the pupil's knowledge and construction of the graph $Z(t)$ the program calculates probability $P_i = p_{i,1} p_{i,2} \ldots p_{i,n}$ of reproduction by him of the $i-$th information block and simulates repeated "testing" of the learner in the moments $t_k = k\Delta t$. The correct answer is simulated as casual process occurring with probability P: the casual variable x which regularly distributed in an interval [0; 1] is generated, also the condition $x < P$ is checked. If the condition is true, it is considered that the learner has answered correctly, and differently – wrong. The pupil's knowledge quantity $Z(t)$ is equal to the number of information blocks, which he can reproduce, and measures in conditional units. At such "testing" the pupil's knowledge does not increase, the probabilities $p_{i,j}$ remain constant. For this modeling the program PR-1 in Free Pascal is used.

Results of modeling

The typical graphs of dependences $Z(t),\ p_a(t),\ \tau_s(t), W(t)$ are submitted in a fig. 1. It is visible, that during training $(0 \le t \le T = 4000)$ the quantities of knowledge of ideas Z and average probability $p_a = z_1/((1-D)N \cdot M)$ for new LME-s at first quickly grow and then reach the maximal meanings $Z_m = N = 50$ and $p_a^{\max} = 1$. The working capacity W decreases on the logistic law. The average time of the tasks solving t_F at first reduces, reaches the minimum (pupil has learned to solve tasks), and then it increases because of downturn of working capacity $W(t)$.

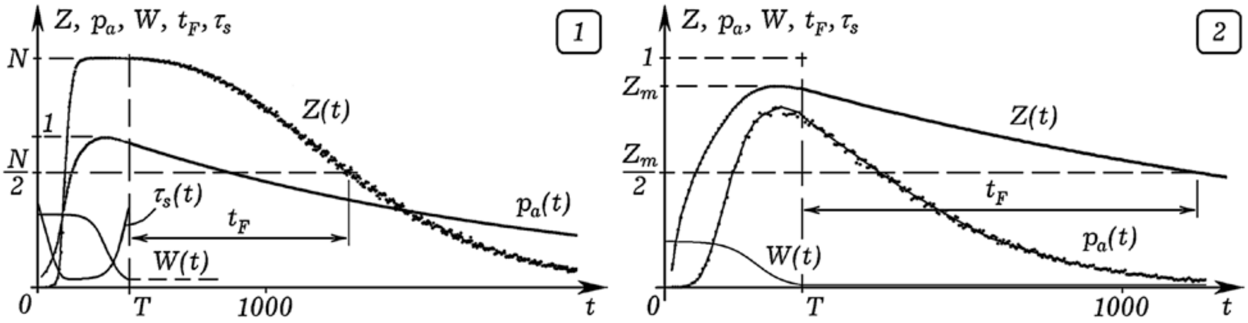

Fig. 1. Training at various coefficient of assimilation ($M = 7,\ N = 50,\ T = 4000$).

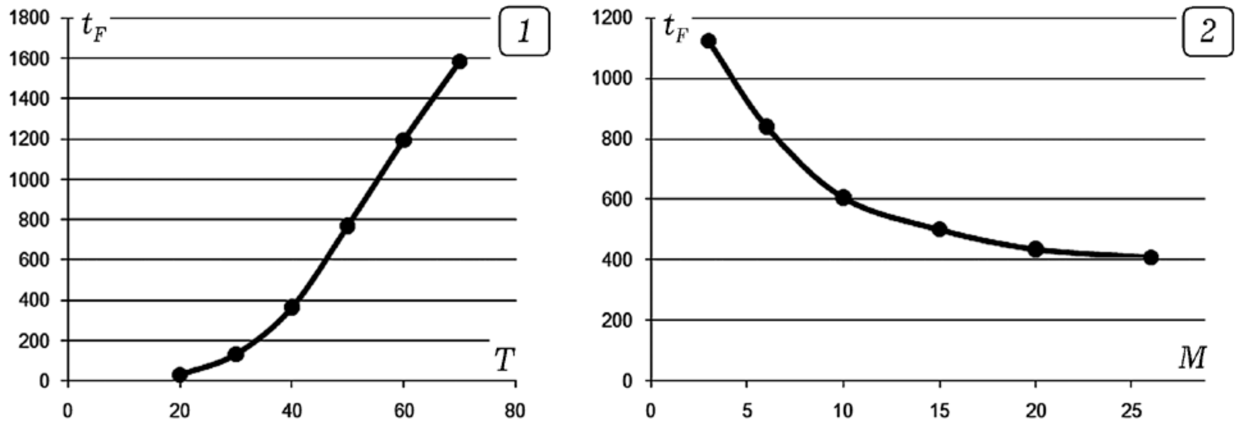

Fig. 2. Dependences of the time of the half information forgetting t_F on T and M .

The dependences of the forgetting time t_F of the half studied information ("semi-forgetting" time) on the duration of lesson T and on the length M of the chain of LME-s has practical importance. If to neglect decrease of working capacity of the pupil because of weariness ($W = 1 = \text{const}$), then with increasing T (at constant $M = 10$) the number of addresses to each LME increases, the strength of assimilation and "semi-forgetting" time t_F grows (fig. 2.1). As the length M of the blocks decreases, the learning material is remembered more firmly, and the forgetting time t_F of the half studied information is growing. Increasing the length of the LME chain at fixed T reduces the probability of the pupil repeating of the corresponding block, lowers the time t_F of forgetting of the half studied information (fig. 2.2). At increase of number of ideas N , studied on a lesson, decrease $s_{i,j}$ and t_F. In this case the quantity of pupil's model addresses to one LME reaches 5–15 thousand, this is in many times more the numbers of addresses to LME of the real learner's during a lesson (3–8 times). It is possible to consider, that $s_{i,j} = 1000$ corresponds to one address of the real pupil to given (i, j) – LME, or decision of one task (answer to one elementary question). It also concerns to variable Z varied in an interval from 0 up to 10^5. The similar change of scale allows to approach to creation of model more flexibly and to receive smooth curves.

If to take into account a change of working capacity W with current of time, the graphs represented in a fig. 3 turn out. At increase M the "semi-forgetting" time t_F decreases to 0, the pupil forgets the studied material faster (fig. 3.1). Let's designate through Z_e the number of acquired information blocks at the end of training. With growth T the knowledge quantity Z_e at first is equal 0, and then raises to some maximal meaning and further remains constant (fig. 3.2). When the duration of a lesson T exceeds $\approx 1{,}1 \cdot \theta$, the learner's working capacity reduces so much, that he does not acquire the information.

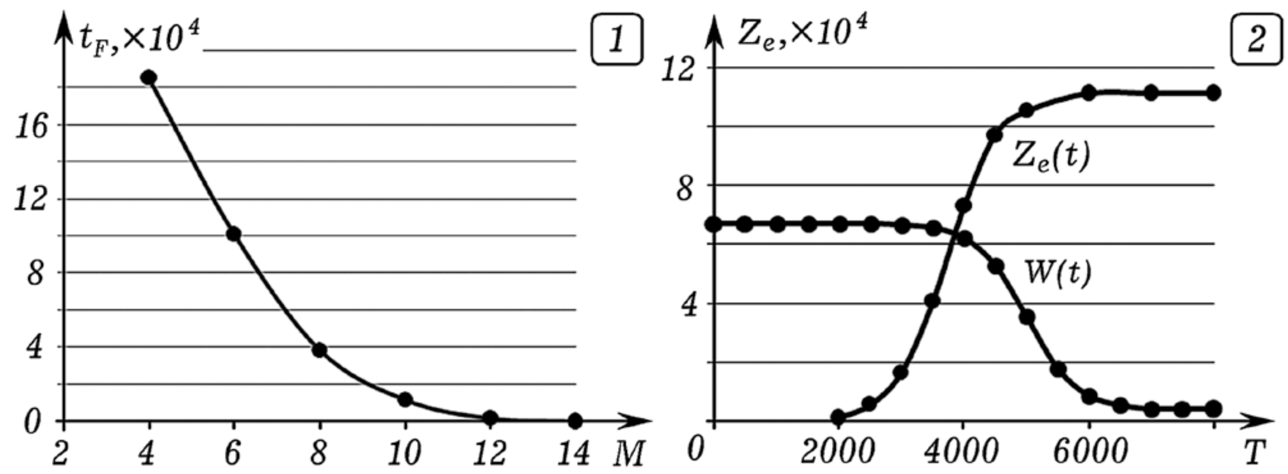

Fig. 3. Results of modeling $T = 3000$. The graphs of dependences $t_F(l)$ and $Z_e(t)$.

Conclusions

In the article it is considered the simulation model of study of the logically connected information, which taking into account change of working capacity of the learner during a lesson. With its help the dependence of durability of knowledge on quantity of ideas N, stated by teacher, their lengths M and duration of a lesson T was studied. The model has allowed to prove the following regularities: 1) the more number N of ideas at $T = \text{const}$ and $M = \text{const}$, the less durability of the acquired knowledge and the "semi-forgetting" time t_F; 2) the more length M of considered ideas at $T = \text{const}$ и $N = \text{const}$, the "semi-forgetting" time t_F is less; 3) at first the increase of the duration of training T at $M = \text{const}$ и $N = \text{const}$ cause a growth of knowledge Z_e at the end of training and rising of "semi-forgetting" time t_F, and after that Z_e and t_F remain practically constant. The quantity and durability of the knowledge, acquired at a lesson, can not surpass critical meaning. It is connected to weariness of the pupil, decreasing of his working capacity and growing of time of the task solving. At use of the model which is not taking into account reduction of the pupil's working capacity, we get that the increase of the lesson duration leads out to the unlimited growth of quantity and durability of studied information (that is Z_e and t_F).

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Abstract

The computer model of mastering of the logically connected material is considered, the dependences of knowledge quantity and forgetting time of half of the acquired information from duration of lesson is investigated. The used approach assumes that the educational material is presented as a set of N separate ideas or information blocks. Each block consists of M learning material elements, ordered and connected with logic links. The probability of the given idea reproduction by the pupil is equal to the product of the probabilities for all LME-s making this idea. To take into account reduction of the pupil's working capacity, we admit that coefficient of assimilation is connected with the duration of training by logistic law. The model has allowed to prove the following regularities: 1) the more number of ideas or their length, the less durability of the acquired knowledge and the “semi-forgetting” time; 2) at first the increase of the duration of training cause a growth of knowledge at the end of training and rising of “semi-forgetting” time, and after that these sizes remain practically constant. The quantity and durability of the knowledge, acquired at a lesson, can not surpass critical meaning. It is connected to weariness of the pupil, decreasing of his working capacity and growing of time of the task solving. The program on Pascal and turned out graphs are presented.

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