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MATHEMATICAL AND COMPUTER MODELING OF ELECTIONS WITH CONSTANT DEMOGRAPHIC FACTOR

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Abstract

In this paper the dynamics of the election process in the case of the transformation of bipartisan elections (the ruling party and one opposition party) into three-party (ruling and two opposition parties) is considered within the inter-election period. Some special case is studied.

In the particular case, with a non-zero demographic factor the exact analytical solution is found. In this case the model parameters are constant in the segment [0, T_1], and in the interval (T_1, T) some relations exist between the constant coefficients of the model ($t = 0$ - is the time of previous elections, $t=T$ is the time of the following elections, $t = T_1 (\ 0 < T_1 < T)$ - time-point, when to political arena there is the second opposition party).

When creating a computer model of the process, coefficients of model are taken as variable time functions. It should be noted that with selection of certain variable coefficients of the model (when the coefficient of attracting supporters for the emerging second opposition party significantly exceeds the coefficients of attracting the other two parties), in the time interval (T_1, T) there is an effect of an increase and a subsequent decrease in the number of supporters of the first opposition and the ruling party, i.e. the functions have a stationary points. The process of the appearance of a new party in the period between elections (in many countries this period is T = 4 years = 1460 days) is considered after 400, 800 and 1000 days from the date of the previous elections. To analyze the results obtained, the case T_1 = 400 days is taken.

Keywords*: two-party, three-party, elections, variable coefficients, analytical solutions, computer modeling.*

Introduction

Mathematical and computer modeling has been widely recognized in disciplines such as sociology, economics, history, political science and others [1].

In [2, 3] quantities of information streams by means of mathematical models of information warfare are studied. By information warfare the authors consider the antagonism by means of mass media between the two states' purposeful misinformation and propagation against each other. The nonlinear mathematical and computer model of information warfare shows that with sufficient activity of the peace-building organizations it is possible to extinguish the expansion of the information warfare.

In [4] the nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutions is offered. The general model shows that irrespective of high aggression of confronting sides, interstate authoritative institutions can extinguish information warfare even when for this purpose efforts of the international organizations are insufficient.

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The current developments in the modern world contributed to the study of and led to the creation of mathematical and computer models of social processes, such as: public administration and administrative management [5-7], demographic and social processes of assimilation [8-10], interference in fundamental and applied researches of scientific research institutes [11-13], problems of training of the diploma scientists [14], problems of linguistic globalization [15].

The mathematical modeling of political elections is of great interest. Many scientists deal with this topic. In most cases, stochastic models of political elections are proposed, which involve an analysis of the statistics of already held elections [16, 17].

The method adopted in [18] of combining empirical analyses with model building, may lead to a better understanding of the nature of representative government under different institutional arrangements and electoral rules.

In [19] a generalized version of the spatial model of electoral competition is proposed. A model of political parties is developed and a general theorem about the existence of distinct Nash equilibrium distributions of party activists is proven. The candidates are assumed to acquire resources from the party and its activists and through the candidate’s own campaign organization to assist in their campaign efforts. Also the formal properties of this more general model is studied, especially the impact of party-based resources.

In [20] the role of strategic motivation in mediating the relationship between underlying political preferences and vote choice is examined. The political preference data taken from the British Election Panel Survey, 1997–2001, was modeled with mixed multinomial logit models. Latent variables were used to model the stable party political traits underlying observed preferences, allowing correlation between choices and so avoiding the restrictive assumption of independence from irrelevant alternatives. Ranked approval ratings were used to characterize the underlying political preferences in the presence of insincere voting. From these models we estimate that approximately 9% of votes cast may have been affected by strategic factors. In keeping with ‘Duvergers law’, the smallest of the three main parties, the Liberal Democrats, were found to be most affected by strategic voting.

Extremely important is the creation of a mathematical model, which would give an opportunity to define the dynamics of change in the number of supporters of different political subjects during the election period and a possible forecast of the election results.

In [21-25] a two-party or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections is considered when coefficients are constant. The assumption was made that the number of voters remains the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received and the conditions under which opposition party can win the upcoming elections were established.

In [26, 27] a two-party (pro-government and opposition parties) nonlinear mathematical model of elections is considered with variable coefficients.

In work [28, 29] the nonlinear mathematical model with variable coefficients in the case of three-party elections is proposed, that describes the dynamics of quantitative change of the votes of the pro-government and two opposition parties from election to election. The model takes into account the change in the total number of voters in the period between two consecutive elections, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In the particular case exact analytical solutions are obtained.

In particular, assumed that in the period between elections coefficients of “attracting” voters are exponentially increasing function of time. In the particular case the exact analytical solutions were obtained.

Of particular interest is the change in the number of political actors between the election [30, 31].

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1. General Mathematical Model of Transformation of Two-party Elections to Three-Party Elections

We will consider the general mathematical model of transformation of two-party elections to three-party elections, from the accounting of demographic factor of elections which we have an appearance:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_2(t))N_1(t)N_2(t) + (\alpha_1(t) - \alpha_3(t))N_1(t)N_3(t) - F_1(N_1(t),t) + \gamma_1(t)N_1(t) \\ \\ \dfrac{dN_2(t)}{dt} = (\alpha_2(t) - \alpha_1(t))N_1(t)N_2(t) + (\alpha_2(t) - \alpha_3(t))N_2(t)N_3(t) - F_2(N_2(t),t) + \gamma_2(t)N_2(t) \\ \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t) - \alpha_1(t))N_1(t)N_3(t) + (\alpha_3(t) - \alpha_2(t))N_2(t)N_3(t) + F_1(N_1,t) + F_2(N_2,t) + \gamma_3(t)N_3(t) \end{cases}$$

$$t \in (0,T], \tag{1.1}$$

$$\begin{aligned} &N_1(0) = N_{10},\ N_2(0) = 0,\ N_3(0) = N_{30},\ N_{30} > N_{10} \\ &N_2(t) \equiv 0,\ F_2(N_2(t),t) \equiv 0,\ t \in [0,T_1),\ T_1 < T \\ &N_2(T_1) = N_{20} > 0 \end{aligned} \tag{1.2}$$

In this nonlinear mathematical model (1.1), (1.2) all coefficients are variables and demographic factors are taken into consideration.

Equations (1.1) is defined in the interval $t \in (0,T]$, and initial conditions (of Cauchy) $t = 0$ moment of time.

We look for the solution of the Cauchy problemon the segment $t \in [0,T]$ in the class of continuous differentiable functions

$$N_1, N_2, N_3 \in C^1[0,T]. \tag{1.3}$$

In a nonlinear system of differential equations (1.1):

$N_1(t), N_2(t), N_3(t) -$ are the numbers of supports of two opposition and one ruling party $N_3(t)$ at time *t*;

$t = 0$ - is the time of previous elections, when one of the parties $N_3(t)$ won the elections and became the ruling party;

$t = T -$ is the time of the following elections (in many cases *T*= 4 years or 1460 days);

$t = T_1$($0 < T_1 < T$) - time-point, when to political arena there is the second opposition party $N_2(t)$;

$\alpha_1(t), \alpha_2(t), \alpha_3(t)$ -- are the coefficients of attracting votes bythe first and second opposition party and the ruling party at time *t*. They largely depend on the action programs, as well as financial, technological and informational capacities of the political parties.

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$F_1(N_1(t),t), F_2(N_2(t),t)$ --are the continuous positive functions, that define the scale of used administrative resources.
$\gamma_1(t),\gamma_2(t),\gamma_3(t)$ -- are the coefficients that describe demographic changes of the parties.

2. A Special Case with Constant Demographic Factor of Elections

We will consider a special case when the functions characterizing use of administrative resources are linear concerning the first variables

$$F_1(N_1(t),t) = \beta_1(t)N_1(t),\ F_2(N_1(t),t) = \beta_2(t)N_2(t), \tag{2.1}$$

and

$$\gamma_1(t) = \gamma_3(t) = \gamma_1 = const \qquad t \in [0, T_1] \tag{2.2}$$

$$\alpha_1(t) = \alpha_1 = const, \alpha_3(t) = \alpha_3 = const$$

Taking into account (2.1), (1.1) and (1.2) will correspond in the following look

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_3(t))N_1(t)N_3(t) - \beta_1(t)N_1(t) + \gamma_1(t)N_1(t) \\ \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t) - \alpha_1(t))N_1(t)N_3(t) + \beta_1(t)N_1(t) + \gamma_3(t)N_3(t) \end{cases} \tag{2.3}$$

$$t \in (0, T_1]$$

$$N_1(0) = N_{10},\ N_3(0) = N_{30},\ N_{30} > N_{10}$$

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_{11}(t) - \alpha_{31}(t))N_1(t)N_3(t) + (\alpha_{11}(t) - \alpha_2(t))N_1(t)N_2(t) - \beta_{11}(t)N_1(t) + \gamma_1(t)N_1(t) \\ \\ \dfrac{dN_2(t)}{dt} = (-\alpha_{11}(t) + \alpha_2(t))N_1(t)N_2(t) + (\alpha_2(t) - \alpha_{31}(t))N_3(t)N_2(t) - \beta_2(t)N_2(t) + \gamma_2(t)N_2(t) \\ \\ \dfrac{dN_3(t)}{dt} = (-\alpha_{11}(t) + \alpha_{31}(t))N_1(t)N_3(t) + (-\alpha_2(t) + \alpha_{31}(t))N_3(t)N_2(t) + \beta_{11}(t)N_1(t) + \beta_2(t)N_2(t) + \gamma_3(t)N_3(t) \end{cases}$$

$$t \in (T_1, T] \tag{2.4}$$

$$N_1(T_1) = N_{11},\ N_2(T_1) = N_{20} > 0, \qquad N_3(T_1) = N_{31}$$

Thus, we get the two Cauchy problem (2.3), (2.4).

From (1.2), (2.1) and (2.3) we get Cauchy problem

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$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1 - \alpha_3) N_1(t) N_3(t) - \beta_1 N_1(t) + \gamma_1 N_1(t) \\ \\ \dfrac{dN_3(t)}{dt} = (\alpha_3 - \alpha_1) N_1(t) N_3(t) + \beta_1 N_1(t) + \gamma_1 N_3(t) \end{cases} \tag{2.5}$$

$$N_1(0) = N_{10},\ N_3(0) = N_{30},$$

The first integral of Cauchy problem (2.5) has the form

$$t \in [0, T_1] \quad N_1(t) + N_3(t) = a \exp(\gamma_1 t). \tag{2.6}$$
$$N_{10} + N_{30} = a$$

Substituting (2.5) into (2.6), for the function $N_1(t)$ we obtain the Bernoulli equation

$$\frac{dN_1(t)}{dt} - [(\alpha_1 - \alpha_3) a \exp(\gamma_1 t) + \gamma_1 - \beta_1] N_1(t) = -(\alpha_1 - \alpha_3) N_1^2(t) \tag{2.7}$$

The solution of the Bernoulli equation, taking into account the initial condition (1.2), has the form

$$N_1(t) = \exp((\gamma_1 - \beta_1)t) \exp[\frac{(\alpha_1 - \alpha_3)a}{\gamma_1}(\exp(\gamma_1 t) - 1)]\{\frac{1}{N_{10}} + \frac{1}{2}(\alpha_1 - \alpha_3)(\gamma_1 - \beta_1)t^2$$
$$- \frac{(\alpha_1 - \alpha_3)a}{\gamma_1} t + \frac{(\alpha_1 - \alpha_3)a}{\gamma_1^2}[\exp(\gamma_1 t) - 1]\}^{-1}. \tag{2.8}$$

Finally, the solution of the Cauchy problem (2.5) has the form

$$N_1(t) = \exp((\gamma_1 - \beta_1)t) \exp[\frac{(\alpha_1 - \alpha_3)a}{\gamma_1}(\exp(\gamma_1 t) - 1)]\{\frac{1}{N_{10}} + \frac{1}{2}(\alpha_1 - \alpha_3)(\gamma_1 - \beta_1)t^2 -$$
$$- \frac{(\alpha_1 - \alpha_3)a}{\gamma_1} t + \frac{(\alpha_1 - \alpha_3)a}{\gamma_1^2}[\exp(\gamma_1 t) - 1]\}^{-1}.$$
$$N_3(t) = a \exp(\gamma_1 t) - \exp((\gamma_1 - \beta_1)t) \exp[\frac{(\alpha_1 - \alpha_3)a}{\gamma_1}(\exp(\gamma_1 t) - 1)]\{\frac{1}{N_{10}} +$$
$$+ \frac{1}{2}(\alpha_1 - \alpha_3)(\gamma_1 - \beta_1)t^2 - \frac{(\alpha_1 - \alpha_3)a}{\gamma_1} t + \frac{(\alpha_1 - \alpha_3)a}{\gamma_1^2}[\exp(\gamma_1 t) - 1]\}^{-1}, \tag{2.9}$$

$$\gamma_1(t) = \gamma_2(t) = \gamma_3(t) = 0 \ \ t \in (T_1, T] \tag{2.10}$$

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$$N_{11} = N_1(T_1) = \exp((\gamma_1 - \beta_1)T_1)\exp[\frac{(\alpha_1 - \alpha_3)a}{\gamma_1}(\exp(\gamma_1 T_1) - 1)]\{\frac{1}{N_{10}} +$$
$$+\frac{1}{2}(\alpha_1 - \alpha_3)(\gamma_1 - \beta_1)T_1{}^2 - \frac{(\alpha_1 - \alpha_3)a}{\gamma_1}T_1 + \frac{(\alpha_1 - \alpha_3)a}{\gamma_1^2}[\exp(\gamma_1 T_1) - 1]\}^{-1},$$

$$N_{31} = N_3(T_1) = a\exp(\gamma_1 T_1) - \exp((\gamma_1 - \beta_1)T_1)\exp[\frac{(\alpha_1 - \alpha_3)a}{\gamma_1}(\exp(\gamma_1 T_1) -$$
$$-1)]\{\frac{1}{N_{10}} + \frac{1}{2}(\alpha_1 - \alpha_3)(\gamma_1 - \beta_1)T_1{}^2 - \frac{(\alpha_1 - \alpha_3)a}{\gamma_1}T_1 + \frac{(\alpha_1 - \alpha_3)a}{\gamma_1^2}[\exp(\gamma_1 T_1) - 1]\}^{-1}. \quad (2.11)$$

From (2.4), (2.10), (2.11) it is easy to receive the first integral of system

$$N_1(t) + N_2(t) + N_3(t) = N_{11} + N_{20} + N_{31} = c, \quad t \in (T_1, T]\,. \quad (2.12)$$

If takes place

$$c(\alpha_{11} - \alpha_2) = \beta_{11} - \beta_2,$$

then

$$N_1(t) = \frac{N_{11}}{N_{20}}N_2(t),\ t \in (T_1, T],$$

$$p = \frac{N_{20}}{N_{11}}.$$

Then the exact solution of the Cauchy problem (2.4) has the form

$$\begin{cases} N_1(t) = \dfrac{\dfrac{(\alpha_{11} - \alpha_{31})c - \beta_{11}}{\alpha_{11} - \alpha_{31} + p(\alpha_2 - \alpha_{31})} N_{11}\exp\{[(\alpha_{11} - \alpha_{31})c - \beta_{11}](t - T_1)\}}{\dfrac{(\alpha_{11} - \alpha_{31})c - \beta_{11}}{\alpha_{11} - \alpha_{31} + p(\alpha_2 - \alpha_{31})} + N_{11}\exp\{[(\alpha_{11} - \alpha_{31})c - \beta_{11}](t - T_1)\} - N_{11}} \\ N_2(t) = pN_1(t) \\ N_3(t) = c - (p+1)N_1(t) \end{cases} \quad (2.13)$$

$$t \in (T_1, T]$$

3. Computer №odeling

For the computer simulation of the process of transforming bipartisan elections into three-party elections in the general case (1.1), the model coefficients are taken as variable time functions. The process of entering the new opposition party's political arena in the period between the two elections for definiteness was considered after 400, 800 and 1200 days from the date of the previous elections. To analyze the results obtained the case T_1 = 400 days is taken**.**

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For a specific numerical count, the coefficients of the model are taken as exponentially increasing functions, although in the sequel other cases can be considered (for example, as exponential functions, periodic functions, etc.).

For clarity, with a numerical calculation, the number of voters was chosen by the example of two countries - Georgia and Australia. As a rule, on election day, the percentage of voters turnout is approximately the same for all parties, so this figure hardly affects the results of elections. The results of the numerical account are conditionally divided into 8 cases.

Case 1:

Until T_1 = 400 days, the number of supporters of the opposition party is decreasing, while the number of supporters of the ruling party is increasing. After this moment until the next election T = 1460 days, the number of supporters of all three parties increases. Similarly, depending on the selection of the model coefficients, two election results were obtained:

1. By the election day the number of supporters of both opposition parties in total exceeds the number of supporters of the pro-government party, i.e. the ruling party loses the election;
2. By the election day, the number of supporters of the pro-government party exceeds the sum of the supporters of both opposition parties, i.e. the ruling party again wins the elections (ex. figure 1).

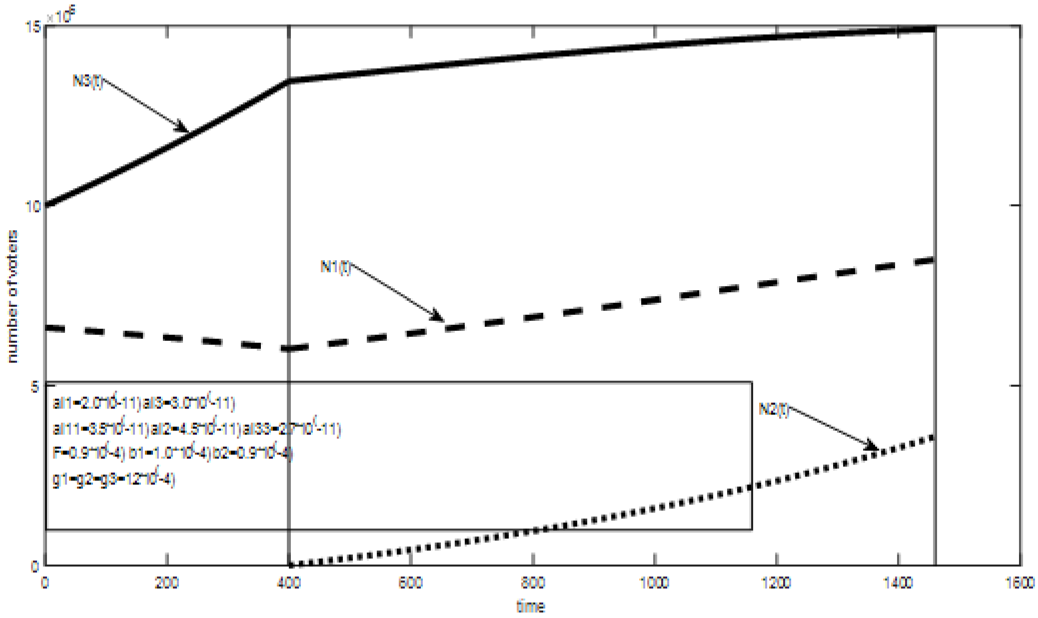

Fig 1.

Case 2:

Up to the moment T_1 = 400 days (when only two parties compete for the involvement of supporters), and after that moment (the third party is on the political arena) until the next election T=1460 days, the number of supporters of all three parties increases. At the same time, depending on the selection of the model coefficients, two election results were obtained:

1. By the election day, the number of supporters of both opposition parties in total exceeds the number of supporters of the pro-government party, i.e. the ruling party loses the elections (ex. figure 2);

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2. By the election day, the number of supporters of the pro-government party exceeds the number of supporters of both opposition parties in the amount, i.e. the ruling party again wins the elections.

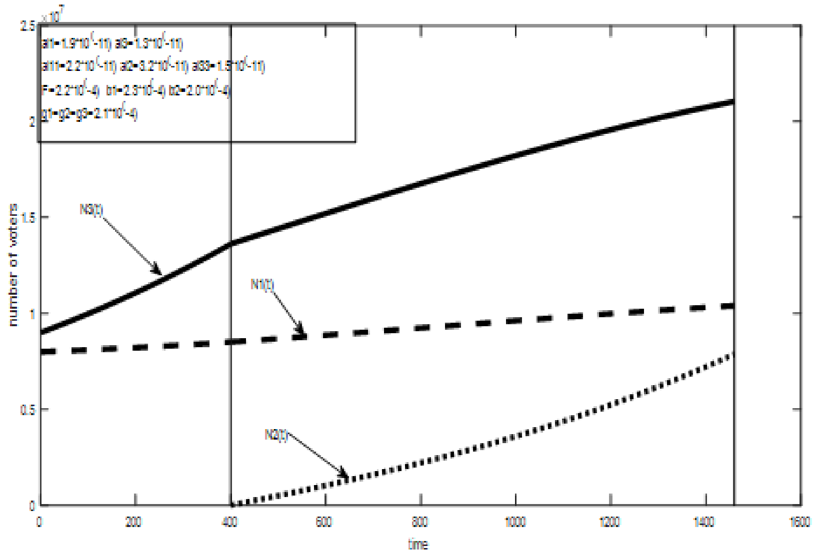

Fig 2.

Case 3:

Until the time T_1 = 400 days the number of supporters of both parties increases. After this moment until the next election T = 1460 days, the number of supporters of the first opposition party decreases, whereas the number of supporters of the second opposition party and the ruling party increases. Depending on the selection of the model coefficients, two election results were obtained:

1. The ruling party loses the election;
2. The ruling party wins the election (ex. figure 3).

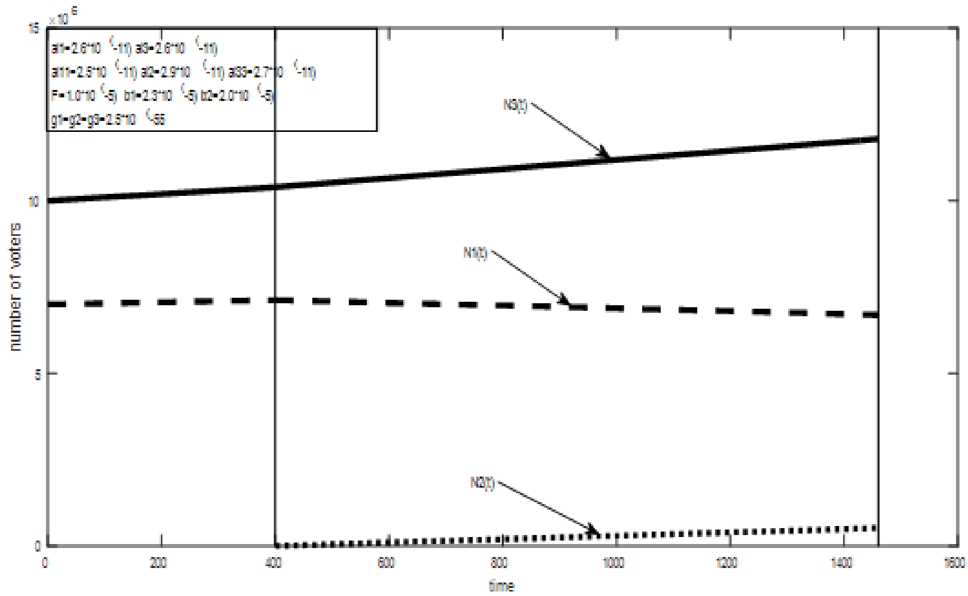

Fig 3.

Case 4:

Until the time T_1 = 400 days the number of supporters of the opposition party is increasing, while the number of supporters of the ruling party is decreasing. After this moment until the next election T = 1460 days, the number of supporters of all three parties increases. Similarly, depending on the selection of the model coefficients, two election results were obtained:

1. By the election day, the number of supporters of both opposition parties in total exceeds the number of supporters of the pro-government party, i.e. the ruling party loses the election (ex. figure 4);
2. By the election day, the number of supporters of the pro-government party exceeds the number of supporters of both opposition parties in the amount, i.e. the ruling party again wins the elections.

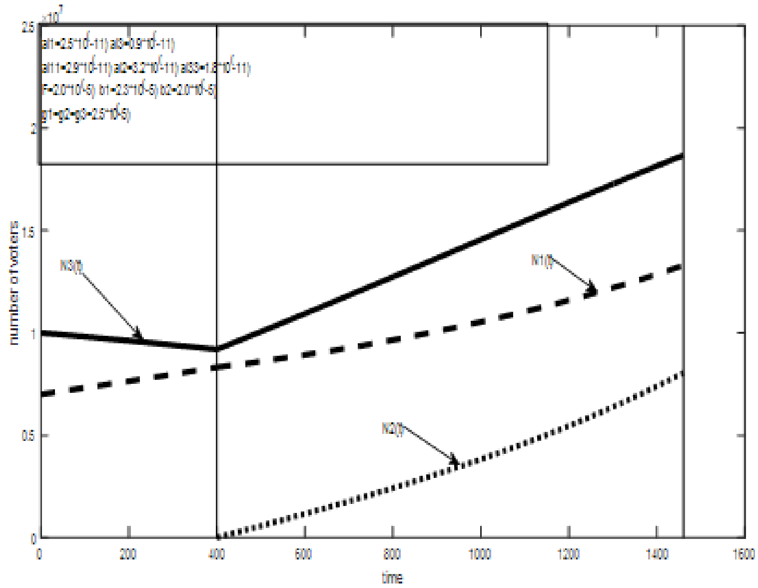

Fig 4.

Case 5:

Until the time T_1 = 400 days the number of supporters of the opposition party is increasing, while the number of supporters of the ruling party is decreasing. After this moment until the next election T = 1460 days, the number of supporters of the first and second opposition parties increases, whereas the number of supporters of the ruling party decreases. Depending on the selection of the model coefficients, two election results were obtained:

1. The ruling party loses the election (ex. figure 5);
2. The ruling party wins the election.

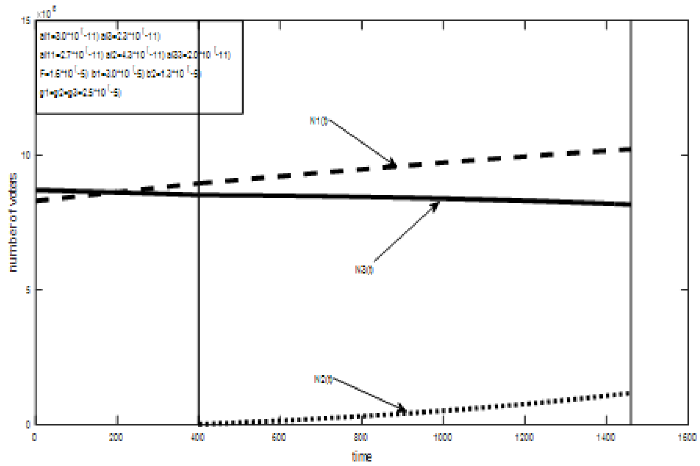

Fig 5.

Case 6:

Until the time T_1 = 400 days the number of supporters of both parties increases. After this moment until the next election T = 1460 days, the number of supporters of the first and second opposition parties increases, whereas the number of supporters of the ruling party decreases. Depending on the selection of the model coefficients, two election results were obtained:

1. The ruling party loses the election (ex. figure 6);
2. The ruling party wins the election.

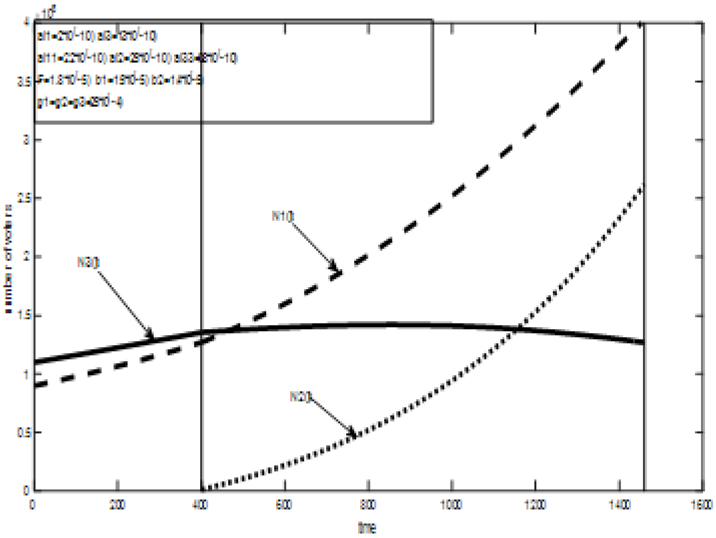

Fig 6.

Case 7:

Until the time T_1 = 400 days, the number of supporters of the opposition party is decreasing, while the number of supporters of the ruling party is increasing. After this moment until the next election T = 1460 days, the number of supporters of the first and second opposition parties increases, whereas the number of supporters of the ruling party decreases. Depending on the selection of the model coefficients, two election results were obtained:

1. The ruling party loses the election;
2. The ruling party wins elections (ex. figure 7).

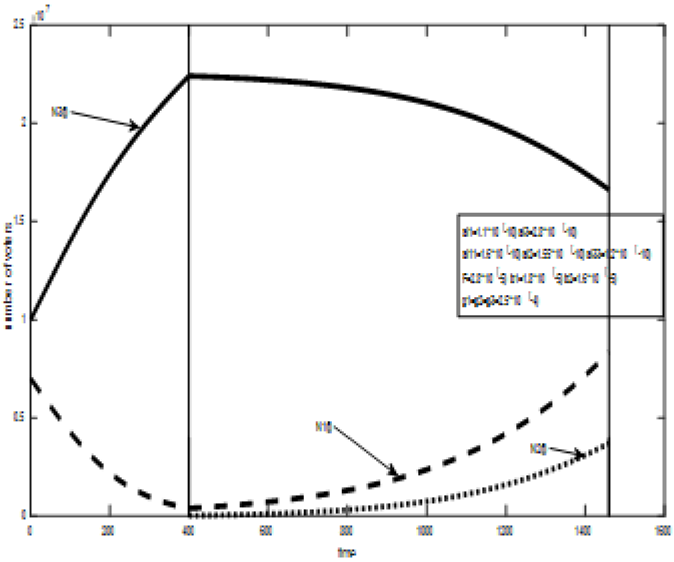

Fig 7.

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Case 8:

Until the time T_1 = 400 days the number of supporters of the opposition party is increasing, while the number of supporters of the ruling party is decreasing. After this moment until the next election T = 1460 days, the number of supporters of the first opposition party decreases, whereas the number of supporters of the second opposition party and the ruling party increases. Depending on the selection of the model coefficients, two election results were obtained:

1. The ruling party loses the election (ex. figure 8);
2. The ruling party wins the elections.

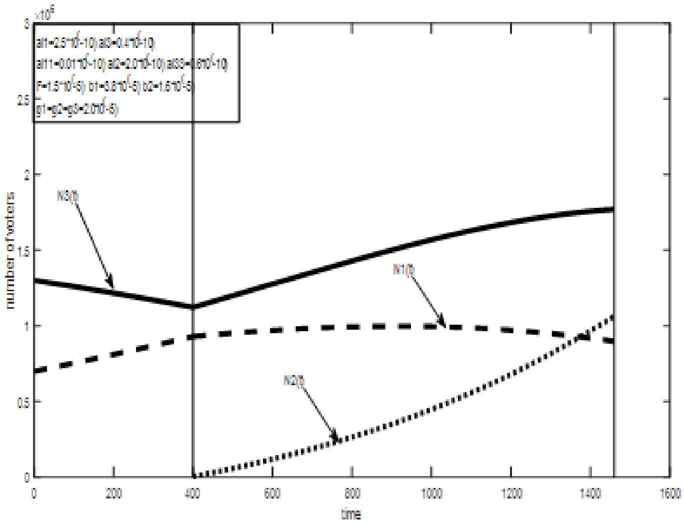

Fig 8.

It should be specially noted that with some selection of the variable coefficients of the model (for example, when the coefficient of attracting supporters for the emerging second opposition party significantly exceeds the coefficients of attracting the other two parties), in the time interval (T_1, T) there is an effect of an increase and a subsequent decrease in the number of supporters of the first opposition and the ruling party, i.e. the functions N_1 (t) and N_3 (t), $t \in (T_1, T)$, have a stationary point $t^* \in (T_1, T)$, where $N_1^{'}(t^*) = 0$ and stationary point $t^{**} \in (T_1, T)$, where $N_3^{'}(t^{**}) = 0$ (figure 9).

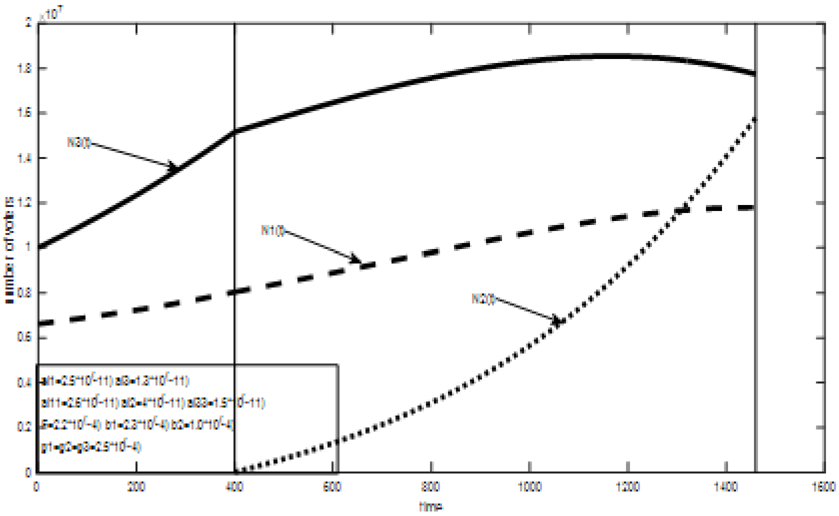

Fig 9.

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Conclusion

The paper considers a non-linear mathematical model of the transition of bipartisan elections to three-party elections, when in the period between elections the second opposition party appears on the political arena. In the particular case, with a non-zero demographic factor, an analytical solution is found when the model parameters are constant in the segment $[0,T_1]$ and in the segment $[T_1,T]$ some relations exist between the coefficients of the model.

The obtained exact solution (2.13) can be used by both the ruling and opposition parties to correct their actions (by actively attracting voters) and achieving the desired result.

In the case of variability of all coefficients of the model, numerous computer calculations were performed, in which, depending on the choice of variable coefficients and initial data, different election results were obtained.

The results of the numerical account can be used by both the ruling and opposition parties to achieve their goals.

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