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RESEARCH OF THE DYNAMIC SYSTEM DESCRIBING MATHEMATICAL MODEL OF SETTLEMENT OF THE CONFLICT BY MEANS OF ECONOMIC COOPERATION

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Abstract.
The paper considers a nonlinear mathematical model of economic cooperation between two politically mutually opposing sides (possibly a country or a country and its subject) that takes into account economic or other type of cooperation between parts of the population of the sides aimed at convergence and peaceful resolution of the conflict. The model implies that the process of economic cooperation is free from political pressure, i.e. the governments of the sides and the third external side does not interfere in this process. A dynamic system has been obtained that describes the dynamics of parts of the population of the sides, focused on cooperation. The model also assumes that both sides have a zero demographic factor, i.e. during the process, the sum of supporters and opponents of cooperation is unchanged. In the case of constancy of the coefficients of the mathematical model, singular points of the nonlinear system of differential equations are found. The problem of stability of solutions is studied. In the case of some dependence between the constant coefficients of the model, the first integral and the exact analytic solution are found. The exact solution obtained allows, within the limits of the given mathematical model and the dependence between its coefficients, to determine the conditions under which economic cooperation can peacefully resolve a political conflict (most of the populations of the sides want conflict resolution).

Keywords: *mathematical model of resolution of conflict, dynamic system, stability of solutions, exact solutions.*

1. Introduction

It is known that mathematical modeling in the social sphere, in sociology, in demography, in the history, in conflictology, in the description of information warfares, processes of assimilation of the people, globalization, etc. was widely adopted in recent years [1 – 14].

In [2] the nonlinear mathematical and computer model of information warfare, with participation of interstate authoritative institutes is offered. Confronting sides in extend of provocative statements, the third side extends of soothing statements, interstate authoritative institutes the peacekeeping statements call the sides for the termination of information warfare. For the general model computer modeling is carried out and shown that irrespective of high aggression of confronting sides, interstate authoritative institutes will be able to extinguish information warfare and when for this purpose efforts of only the international organizations insufficiently.

The current developments in the modern world contributed to study and led to the creation of mathematical and computer models social processes, such as: public administration and administrative management [15], demographics and social processes of assimilation [3, 16, 17], of interference in fundamental and applied researches of scientifically-research institutes [10 - 12], problem of training of the diplomaed scientists [13, 14], problems of linguistic globalization [5].

The mathematical modeling of political elections is of great interest. Many scientists deal with this topic. In most cases, stochastic models of political elections are proposed, which involve an analysis of the statistics of already held elections [18 - 20].

Extremely important is the creation of a mathematical model, which would give an opportunity to define the dynamics of change in the number of supporters of different political subjects during the election period and a possible forecast of the election results.

In [21 - 24] a two-party or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections is considered when coefficients are constant. The assumption was made that the number of voters remains the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received and the conditions under which opposition party can win the upcoming elections were established.

In [24] a two-party (pro-government and opposition parties) nonlinear mathematical model of elections is considered with variable coefficients.

In work [25] the nonlinear mathematical model with variable coefficients in the case of three-party elections is proposed, that describes the dynamics of quantitative change of the votes of the pro-government and two opposition parties from election to election. The model takes into account the change in the total number of voters in the period between two consecutive elections, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In the particular case exact analytical solutions are obtained.

In particular, assumed that in the period between elections coefficients of “attracting” voters are exponentially increasing function of time. In the particular case the exact analytical solutions were obtained.

Of particular interest is the change in the number of political actors between the election [26].

It is known that mathematicians Robert Aumann and Thomas Schelling in 2005 got the Nobel Prize on economy for a cycle of scientific works "Understanding of problems of the conflict and cooperation with the help of the analysis within game theory". Robert Aumann in the lecture when receiving an award noted on December 8, 2005 that: "Wars and other conflicts are one of the main sources of human sufferings. So, it is expedient to devote this lecture of one of the most vital and deep issues facing mankind: war and peace", and Thomas Schelling emphasized that: "The most fascinating event of last half of the century is that which did not occur. We enjoyed sixty years without the nuclear weapon blown in anger".

One of authors of Singapure "economic miracle" Lee Kuan Yew is sure that "If you need economic growth, then it is better not to be at war with neighbors, and to trade".

2.1. Mathematical model describing of settlement of the bilateral conflict by means of economic cooperation

The new nonlinear mathematical model (the dynamic system) of economic cooperation between two warring sides offered by us has an appearance:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = -\alpha_1(t)(a - N_1(t))(b - N_2(t)) + \beta_1(t)N_1(t)N_2(t) \\ \dfrac{dN_2(t)}{dt} = -\alpha_2(t)(a - N_1(t))(b - N_2(t)) + \beta_2(t)N_1(t)N_2(t) \end{cases} \tag{1.1}$$

$$N_1(0) = N_{10},\ N_2(0) = N_{20}, \tag{1.2}$$

where $N_1(t)$ - number of the citizens of the first side in time-point t, wishing or already being in economic cooperation and inclined to the subsequent peaceful resolution of the conflict, $N_2(t)-$number of the citizens of the second side in time-point t, wishing or already being in economic cooperation and inclined to the subsequent peaceful resolution of the conflict, $\alpha_1(t),\alpha_2(t)-$coefficients of aggression (alienation) of the sides, $\beta_1(t),\beta_2(t)-$coefficients of cooperation of the sides, $a,b-$ the population according to the first and second sides which in model are assumed invariable (zero demographic factor), $N_1,N_2\in C^1[0,T]$, $T-$time-point of resolution of conflict.

We assume that rather weak (1.3) and strong (1.4) conditions of resolution of conflict are:

$$\begin{cases}\frac{a}{2}<N_1(t)\le a\\ \frac{b}{2}<N_2(t)\le b\end{cases} \qquad t\ge t_* \tag{1.3}$$

$$\begin{cases}\frac{2a}{3}\le N_1(t)\le a\\ \frac{2b}{3}\le N_2(t)\le b\end{cases}. \qquad t\ge t_{**} \tag{1.4}$$

2.2. Special case of constant coefficients of model

Let's consider a special case of constant coefficients of model.

$$\alpha_1(t)=\alpha_1=const>0,\ \alpha_2(t)=\alpha_2=const>0, \tag{2.1}$$

$$\beta_1(t)=\beta_1=const>0,\ \ \beta_2(t)=\beta_2=const>0,$$

The system of the nonlinear differential equations (1.1), in case of (2.1) has the following stationary (special) points:

$M_1(a;0)$, which when performing inequality $\alpha_1\beta_2-\alpha_2\beta_1>0$ is unstable knot, and when performing opposite inequality $\alpha_1\beta_2-\alpha_2\beta_1<0$ - a hyperbolic saddle;
$M_2(0;b)$ - when performing inequality $\alpha_1\beta_2-\alpha_2\beta_1<0$ is unstable knot, and when performing opposite inequality $\alpha_2\beta_1-\alpha_1\beta_2<0$ - hyperbolic saddle.

When performing equality $\alpha_1\beta_2-\alpha_2\beta_1=0$ in the first quarter of the phase plane of solutions we will receive a curve which each point is special point

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$$M_3(q;\frac{\alpha_1 b(a-q)}{\beta_1 q+\alpha_1(a-q)}), \qquad 0<q<a \tag{2.2}$$

Let's assume that the ratio takes place

$$\alpha_1\beta_2-\alpha_2\beta_1=0, \qquad \frac{\alpha_1}{\beta_1}=\frac{\alpha_2}{\beta_2}=p \tag{2.3}$$

Then the system (1.1), taking into account (2.1), (2.2) will correspond in the following look

$$\begin{cases} \dfrac{dN_1(t)}{dt}=-\beta_1[p(a-N_1(t)(b-N_2(t))-N_1(t)N_2(t)] \\ \dfrac{dN_2(t)}{dt}=-\beta_2[p(a-N_1(t))(b-N_2(t))-N_1(t)N_2(t)] \end{cases} \tag{2.4}$$

From (2.4), taking into account initial conditions (1.2), it is easy to receive the first integral of system of the differential equations

$$N_2(t)=N_{20}+\frac{\beta_2}{\beta_1}(N_1(t)-N_{10}) \tag{2.5}$$

Further, substituting (2.5) in the first equation (2.4), for $N_1(t)$ function we will receive the following differential equation

$$\frac{dN_1(t)}{dt}=pa(-\beta_1 b+\beta_1 N_{20}-\beta_2 N_{10}) + [\beta_1 pb+\beta_2 pa+\beta_1(1-p)N_{20}-(1-p)\beta_2 N_{10}]N_1(t)+ +(1-p)\beta_2 N_1^2(t) \tag{2.6}$$

with an initial condition (1.2).

Let's consider some special cases.

1. $p=1$

Then the solution of a task of Cauchy (2.6), (1.2) has an appearance

$$N_1(t) = \frac{1}{\beta_1 b + \beta_2 a}[\beta_1(bN_{10} + aN_{20} - ab)\,exp[(\beta_1 b + \beta_2 a)t] + a(\beta_1(b - N_{20}) + \beta_2 N_{10})] \quad (2.7)$$

The analysis of the solution (2.7), shows that at

$$bN_{10} + aN_{20} < ab \quad (2.8)$$

inequalities take place

$$\frac{dN_1(t)}{dt} < 0, \qquad \frac{dN_2(t)}{dt} < 0, \quad (2.9)$$

at

$$bN_{10} + aN_{20} = ab \quad (2.10)$$

equalities take place

$$\frac{dN_1(t)}{dt} = 0, \qquad \frac{dN_2(t)}{dt} = 0, \quad (2.11)$$

at

$$bN_{10} + aN_{20} > ab \quad (2.12)$$

inequalities take place

$$\frac{dN_1(t)}{dt} > 0, \qquad \frac{dN_2(t)}{dt} > 0 \quad (2.13)$$

From (2.7) it is easy to show that at (2.12) equality is right

$$N_1(t_*) = a, \;\; t_* = \frac{1}{\beta_1 b + \beta_2 a}\ln\frac{a[\beta_1 N_{20} + \beta_2(a - N_{10})]}{\beta_1(bN_{10} + aN_{20} - ab)} \quad (2.14)$$

$$N_2(t_*) = N_{20} + \frac{\beta_2}{\beta_1}(a - N_{10})$$

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2. $p \neq 1$

Let's enter designations

$$\delta^2 \equiv pa[\beta_2 N_{10} + (b - N_{20})\beta_1] \tag{2.15}$$

$$\varepsilon \equiv \beta_1 pb + \beta_2 pa - \beta_1 (p-1)N_{20} + (p-1)\beta_2 N_{10} \tag{2.16}$$

In the first case

$$\varepsilon^2 = 4\delta^2 (p-1)\beta_2, \tag{2.17}$$

what is possible only at $p > 1$, the solution (2.6), (1.2) has an appearance

$$N_1(t) = \frac{N_{10} + \frac{\varepsilon}{2}[N_{10} - \frac{\varepsilon}{2(p-1)\beta_2}]t}{[(p-1)\beta_2 N_{10} - \frac{\varepsilon}{2}]t + 1} \tag{2.18}$$

at the same time, it agrees (2.16), at $p > 1$, $\varepsilon > 0$

$$\lim_{t \to \infty} N_1(t) = \frac{\varepsilon}{2(p-1)\beta_2} \tag{2.19}$$

In the second case

$$\varepsilon^2 < 4\delta^2 (p-1)\beta_2, \tag{2.20}$$

what perhaps only at $p > 1$, the decision (2.6) has an appearance

$$N_1(t) = \frac{\varepsilon}{2(p-1)\beta_2} + q\frac{N_{10} - \frac{\varepsilon}{2(p-1)\beta_2} - q\tan((p-1)q\beta_2 t)}{q + (N_{10} - \frac{\varepsilon}{2(p-1)\beta_2})\tan((p-1)q\beta_2 t)} \tag{2.21}$$

at the same time, it agrees (2.16), at $p > 1$, $\varepsilon > 0$.

In the third case

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$$\varepsilon^2 > 4\delta^2(p-1)\beta_2 \tag{2.22}$$

$$N_{11} = \frac{\varepsilon}{2(p-1)\beta_2} - \frac{\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}}{2|p-1|\beta_2} \tag{2.23}$$

$$N_{12} = \frac{\varepsilon}{2(p-1)\beta_2} + \frac{\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}}{2|p-1|\beta_2}$$

$$N_1(t) = \frac{N_{12}(N_{10} - N_{11}) - N_{11}(N_{10} - N_{12})\exp\{t\frac{(1-p)}{|p-1|}\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}}{N_{10} - N_{11} + (N_{12} - N_{10})\exp\{t\frac{(1-p)}{|p-1|}\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}} \tag{2.24}$$

If $p > 1$, then

$$N_1(t) = \frac{N_{12}(N_{10} - N_{11}) - N_{11}(N_{10} - N_{12})\exp\{-t\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}}{N_{10} - N_{11} + (N_{12} - N_{10})\exp\{-t\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}} \tag{2.25}$$

$$\lim_{t\to\infty} N_1(t) = \frac{\varepsilon + \sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}}{2(p-1)\beta_2} \tag{2.26}$$

If $p < 1$

$$N_1(t) = \frac{N_{12}(N_{10} - N_{11}) - N_{11}(N_{10} - N_{12})\exp\{t\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}}{N_{10} - N_{11} + (N_{12} - N_{10})\exp\{t\sqrt{\varepsilon^2 - 4\delta^2(p-1)\beta_2}\}} \tag{2.27}$$

$$\lim_{t\to\infty} N_1(t) = \frac{\varepsilon + \sqrt{\varepsilon^2 + 4\delta^2(1-p)\beta_2}}{2(1-p)\beta_2} \tag{2.28}$$

The analysis of the received exact solution (2.7) – (2.28) allows within this mathematical model and some dependences between its coefficients, to define conditions at which economic cooperation will be able peacefully to resolve a political conflict (most of the population of the sides wish resolution of conflict).

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3. Conclusion

Thus, from the analysis of the received analytical solutions within the offered mathematical model, it is possible to define conditions on model parameters (the number of the population of the sides; initial values of supporters of economic cooperation; coefficients of aggression of the sides; coefficients of economic cooperation of the sides) at which resolution of conflict by means of economic cooperation between politically warring sides is possible.

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