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STABLE PLASMA STRUCTURES AS AN ALTERNATIVE ENERGY SOURCE (METHOD FOR SOLVING THE NONLINEAR SCHRÖDINGER EQUATION)

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Abstract

In this paper investigates the spectral problem for the nonlinear Schrödinger equation. The initial infinite region is replaced by a finite region where the grid is constructed and on this grid the Schrödinger equation is reduced to finite difference equations. To solve the spectral problem for the resulting finite-difference equation, the shooting method is used.

Keywords: *nonlinear Schrödinger equation, plasma oscillations, electric dipole moment, stable atmospheric plasma structure, Langmuir soliton.*

1. Introduction and Nonlinear oscillations in plasma. If we study electrostatic plasma oscillations, i.e. when the magnetic field is zero, $\vec{B}=0$, the motion of the electron component of plasma obeys the following plasma hydrodynamics equations:

$$\frac{\partial n_e}{\partial t}+\Delta\cdot(n_e V_e)=0,\quad \frac{\partial V_e}{\partial t}+(V_e\Delta)V_e=-\frac{e}{m}\vec{E}-\frac{1}{n_e m}\Delta p, \tag{1}$$

where n_e is the number density of electrons, V_e is the electron velocity, m is mass of an electron, $e>0$ is the absolute value of the elementary charge, $\vec{E}$ is the strength of the electric field, and p is the pressure We should also consider Maxwell and Poisson equations for the electric field evolution,

$$\frac{\partial \vec{E}}{\partial t}=4\pi e\left(n_e\vec{V}_e-n_i\vec{V}_i\right),\quad \left(\Delta\cdot\vec{E}\right)=-4\pi e\left(n_e-n_i\right), \tag{2}$$

where n_i is the ion number density and $\vec{V}_i$ is ion velocity.

In the zeroth approximation only electrons participate in a plasma oscillation, with the number density of ions being approximately constant $n_i\approx n_0=const$. Thus we may present the electric field in the form,

$$\vec{E}=\vec{E}_1 e^{-i\omega_e t}+\vec{E}_1^* e^{i\omega_e t}+\cdots, \tag{3}$$

where $\omega_e=\sqrt{4\pi e^2 n_0/m}$ is the Langmuir frequency for electrons and $\vec{E}_1$ is the amplitude of the electric field. It should be noted that, in the following, we shall study axially and spherically symmetric plasma oscillations. In this case one can find a scalar potential ϕ_1 such as $\vec{E}_1=-\nabla\phi_1$ in Eq.(3).

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In a realistic situation ions will also participate in a plasma oscillation. Thus the ion density becomes $n_i = n_0 + n(\vec{r}, t)$, where n is the perturbation of the ion density. It leads to the appearance of higher harmonics omitted in Eq.(3). The plasma hydrodynamic equations for the description of the ions evolution have the form[1],

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \left(n_i \vec{V}_i\right) = 0, \quad \frac{\partial \vec{V}_i}{\partial t} + \left(\vec{V}_i \nabla\right)\vec{V}_i = \frac{\vec{F}_i}{M}, \tag{4}$$

where M is the ion mass, $\vec{F}_i$ is the force acting of ions. The reason why one can omit the ion pressure term in Eq. (4) will be discussed bellow.

Using the quasineutrality of plasma we can find that [2]

$$n_0 + n \approx n_e = n_0 \exp\left(\frac{e\varphi_s - U_{pm}}{T_e}\right) \tag{5}$$

where φ_s is the slowly varyng part of the electric potential, T_e is the electron temperature, and $U_{pm} = \left|\vec{E}_1\right|^2 / \left(4\pi n_0\right)$ is the potential of the ponderomotive force which acts on a charged particle in a rapidly oscillating electric field given in Eq. (3). Supposing that ions are mainly involved in the slow motion of plasma we get that $\vec{F}_i = -e\nabla\varphi_s$. Finally, using Eqs. (1)-(5) one arrives to the system of Zakharov equations[2]. More detailed derivation of the nonlinear plasma evolution equations which include electron - ion and electron interactions can be found in Ref.[1].

It should be noted that Eq. (4) is derived under the assumption of ions having point-like charges. However realistic atmospheric plasma contains mainly nitrogen or oxygen ions, which are diatomic. In this case, the simplified ion hydrodynamics Eq. (4) is incomplete since it does not take into account the internal structure of ions.

A diatomic molecule is nonpolar, i.e. it cannot have an intrinsic EDM because of the symmetry reasons. Novertheless, this kind of molecules can acquire EDM, $p_i = \alpha_{ijj} E_j$, in an external electric field. Here $\left(\alpha_{ij}\right)$ is the polarizability tensor. Hence the additional force, $\vec{F}_{pol} = \left(\vec{p}\nabla\right)\vec{E}$, will act on this particle placed in an external inhomogeneous electric field. Thus, if we study the plasma with diatomic ions, in Eq. (4) one should replace $\vec{F}_i = -e\nabla\varphi_s \to -e\nabla\varphi_s + \vec{f}_{pol} / n_i$, where $\vec{f}_{pol}$ is the volume density of ponderomotive force related to the matter polarization.

If an ion is diatomic and possesses an axial symmetry, one can always reduce the polarizability tensor to the diagonal form, $\left(\alpha_{ij}\right) = diag\left(\alpha_\perp, \alpha_\perp, \alpha_\parallel\right)$, where $\alpha_\perp$ and $\alpha_\parallel$ are transversal and longitudinal polarizabilities. The expression for $\vec{f}_{pol}$ has the form[3],

$$\vec{f}_{pol} = \frac{1}{8\pi}\left[\nabla\left(n_i \frac{\partial \varepsilon}{\partial n_i}\vec{E}^2\right) - \vec{E}^2 \nabla\varepsilon\right] = n_i\left[\langle\alpha\rangle + \frac{4}{45}\frac{\left(\Delta\alpha\vec{E}\right)^2}{T_i}\right]\nabla\vec{E}^2, \tag{6}$$

where ε is the permittivity of the ion component of plasma, T_i is the ion temperature, $\langle\alpha\rangle = \left(2\alpha_\perp + \alpha_\parallel\right)/3$ is the mean polarizability of an ion, and $\Delta\alpha = \alpha_\parallel - \alpha_\perp$. It should be noted that the general expression for the ponderomotive force $\vec{f}_{pol}$ was derived under the assumption of static fields with $\left(\nabla \times \vec{E}\right) = 0$. However, as we mentioned above, we study electrostatic plasma oscillations with zero magnetic field. Thus Eq. (6) remains valid.

Combining Eqs. (1)-(6) we get the following nonlinear coupled equations for the amplitude of the electric field,

$$i\dot{\vec{E}}_1 + \frac{3}{2}\omega_e r_D^2 \nabla\left(\nabla \cdot \vec{E}_1\right) - \frac{\omega_e}{2n_0} n\vec{E}_1 = 0, \tag{7}$$

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and for the perturbation of the ion density,

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\nabla^2\right)\cdot n = \frac{1}{4\pi M}\nabla^2\left|\vec{E}_1\right|^2 - \frac{4}{15}\frac{(\Delta\alpha)^2 n_0}{MT_i}\nabla^2\left|\vec{E}_1\right|^4, \tag{8}$$

where $r_D = \sqrt{T_e / 4\pi e^2 n_0}$ is the Debye length and $c_s = \sqrt{T_e / M}$ is the should velocity in plasma. To derive Eq. (8) we take into account that $\langle \vec{E}^4 \rangle = 6\left|\vec{E}_1\right|^4$ while averaging over the time interval $\sim 1/\omega_e$.

The first quadratic term in the rhs of Eq. (8) corresponds to the direct interaction of charged ions with the electric field whereas the second quartic term there, $\sim \nabla^2\left|\vec{E}_1\right|^4$, is related to the induced EDM interaction. Hence the contribution of this second term to the Langmuir waves dynamics should be typically smaller. However, as shown in Ref.[4], in some cases it is the EDM term which arrests the collapse of Langmuir waves.

It should be noted that in Eq.(8) we neglect the contribution of the ion temperature to the sound velocity. Such a contribution would correspond to a nonzero ion pressure term in Eq. (4). Since we suppose that $T_i \langle\langle T_e$, we can omit the ion pressure. However we keep the ion temperature in the quartic nonlinear term in Eq. (8). In the rhs of Eq.(8) we also neglect term $\sim -n_0\langle\alpha\rangle\nabla^2\left|\vec{E}_1\right|^2/M$ which is small compared to the contribution of the Miller force. Indeed, the ratio of these terms is $\sim n_0\langle\alpha\rangle$. We shall use the following values of n_0 and $\langle\alpha\rangle$. We shall use the following values of n_0 and $\langle\alpha\rangle$: $n_0 \sim 10^{21} cm^{-3}$ and $\langle\alpha\rangle \sim 10^{-24} cm^{-3}$. For such a parameters, this ratio $\sim 10^{-3}$, that justifies the validity of Eq.(8).

2. Cubic-quintic nonlinear Schrödinger equation. Let us suggest that the density variation in Eq.(8) is slow, i.e. $\partial^2 n/\partial t^2 << c_s^2\nabla^2 n$. In tis subsonic regime Eqs. (7) and (8) can be cast in a single nonlinear Schrödinger equation (NLSE),

$$i\dot{\vec{E}} + \frac{3}{2}\omega_e r_D^2\nabla\left(\nabla\cdot\vec{E}\right) + \frac{\omega_e}{T_e}\left(\frac{1}{8\pi n_o}\left|\vec{E}\right|^2 - \frac{2(\Delta\alpha)^2}{15T_i}\left|\vec{E}\right|^4\right)E = 0, \tag{9}$$

which has both cubic and quintic nonlinear terms. Note that in Eq.(9) we omit the index “1” in the amplitude of the electric field, i.e. $\vec{E}_1 \equiv \vec{E}$, in order not to encumber the formulas.

We shall examine axially or spherically symmetric plasma oscillations, i.e. $\vec{E} = E\vec{e}_r$, where $\vec{e}_r$ is a unit vector in radial direction and E is a scalar function. Introducing the following dimensionless variables:

$$\tau = \frac{15}{128\pi^2}\frac{T_i}{T_e}\frac{1}{(n_0\Delta\alpha)^2}\omega_e t, \quad x = \frac{1}{8\pi n_0\Delta\alpha}\sqrt{\frac{5T_i}{T_e}}\frac{r}{r_D} \qquad \psi = 4\Delta\alpha\sqrt{\frac{\pi n_0}{15T_i}}E \tag{10}$$

we can represent Eq. (9) in the form [4],

$$i\frac{\partial\psi}{\partial\tau} + \psi'' + \frac{d-1}{x}\psi' - \frac{d-1}{x^2}\psi + \left(|\psi|^2 - |\psi|^4\right)\psi = 0, \tag{11}$$

which contains no dimensionless parameters. Here $d = 2,3$ is the dimension of space. Eq.(11) with boundary conditions: $\psi(0,\tau) = \psi(\infty,\tau) = 0$.

One can check by the direct calculation that the Plasmon number,

$$N = \int_0^\infty \Omega_d dx\cdot x^{d-1}|\psi|^2, \tag{12}$$

and the Hamiltonian

$$H = \int_0^\infty \Omega_d dx\cdot x^{d-1}\cdot\left\{\left|\frac{1}{x^{d-1}}\left(x^{d-1}\psi\right)'\right|^2 - \frac{1}{2}|\psi|^4 + \frac{1}{3}\left|\psi^6\right|\right\} \tag{13}$$

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are the integrals of Eq. (11). Here $\Omega_2 = 2\pi$ and $\Omega_3 = 4\pi$ are the solid angles in two and three dimensions.

Eq. (11) can be applied to model a rare natural atmospheric plasma phenomenon called a *ball lighting* (BL) [5]. There are indications that some BL can be rather powerful energy sources. Thus, if one succeeds to implement a BL in a laboratory, this plasma object can be an alternative source [6].

3. The algorithms for solution NLSE. To the study of such equations are devoted numerous works (see [7,8] and the literature cited there). In some special cases, one can obtain exact solutions of these equations. However, in the general case, finding the exact solution of such an equation is not possible, which forces us to turn to numerical methods. Below we propose a solution to the NLSE with partial derivatives in the form $\psi(x,\tau) = e^{i\lambda\tau}\psi_0(x)$, which leads to the ordinary Schrödinger differential equation with a spectral parameter λ.

For constructing the algorithms for solving Eqs (11)

$$\psi(x;\tau) = e^{i\lambda\tau}\psi_0(x). \tag{14}$$

we will require the following initial and boundary conditions:

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$$\psi(x=0;\tau) = \psi(x=\infty;\tau) = 0, \tag{15}$$

$$\psi(x;\tau=0) = \psi_0(x). \tag{16}$$

Using the obvious following expressions:

$$i\frac{\partial\psi}{\partial\tau} = i\cdot i\lambda e^{i\lambda\tau}\psi_0(x) = -\lambda e^{i\lambda\tau}\psi_0(x)$$

$$\psi'(\lambda) = e^{i\lambda\tau}\psi_0'(x)$$

$$\psi''(\lambda) = e^{i\lambda\tau}\psi_0''(x)$$

$$|\psi|^2 = \left|e^{i\lambda\tau}\psi_0(x)\right|^2 = |\psi_0(x)|^2$$

$$|\psi|^4 = \left|e^{i\lambda\tau}\psi_0(x)\right|^4 = |\psi_0(x)|^4$$

Let us rewritten Eqs. (11) as,

$$\psi_0''(x) + \frac{d-1}{x}\psi_0'(x) - \lambda\psi_0(x) - \frac{d-1}{x^2}\psi_0(x) + \left(|\psi_0|^2 - |\psi_0|^4\psi_0(x)\right) = 0. \tag{17}$$

Considering the consequences of the boundary conditions (14) and (15)

$$\left.\begin{array}{l} \psi_0(0) = 0 \\ \lim\limits_{x\to\infty}\psi_0(x) = 0 \end{array}\right\}, \tag{18}$$

in solution (14), taking into account (12) and (13) in the case of a condition $\psi_0(L) = 0$, the solution of problem (17) - (18) is already in the region $G = \{0 \le x \le L\}$.

Approximating the following

$$\psi_0(x_i) = \psi_0^i$$

$$\psi_0'(x_i) \approx \frac{\psi_0(x_{i+1}) - \psi_0(x_i)}{h} = \frac{\psi_0^{i+1} - \psi_0^i}{h}$$

$$\psi_0''(x_i) \approx \frac{\psi_0(x_{i+1}) - \psi_0(x_i) + \psi_0(x_{i-1})}{h^2} = \frac{\psi_0^{i+1} - 2\psi_0^i + \psi_0^{i-1}}{h^2}$$

and substituting them in Eqs.(17) for the points $x = x_i$ we get the following finite difference equation:

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$$\frac{\psi_0^{i+1}-2\psi_0^i+\psi_0^{i-1}}{h^2}+\frac{d-1}{x_i}\frac{\psi_0^{i+1}-\psi_0^i}{h}-\lambda\psi_0^i-\frac{d-1}{(x_i)^2}\psi_0^i+\left(\left|\psi_0^i\right|^2-\left|\psi_0^i\right|^4\right)\psi_0^i=0\,, \tag{19}$$

$$\psi_0^1=0\,,\ \psi_0^{N+1}=0\,. \tag{20}$$

Grouping the terms by ψ_0^i we get:

$$\psi_0^{i+1}\left[\frac{1}{h^2}+\frac{d-1}{hx_i}\right]+\psi_0^i\left[-\frac{2}{h^2}-\frac{d-1}{hx_i}-\frac{d-1}{(x_i)^2}+\left(\left|\psi_0^i\right|^2-\left|\psi_0^i\right|^4\right)\right]+\psi_0^{i-1}\frac{1}{h^2}=\lambda\psi_0^i.$$

Next, introducing the notation

$$a_i=\left[\frac{1}{h^2}+\frac{d-1}{hx_i}\right],$$

$$b_i=\left[-\frac{2}{h^2}-\frac{d-1}{hx_i}-\frac{d-1}{(x_i)^2}+\left(\left|\psi_0^i\right|^2-\left|\psi_0^i\right|^4\right)\right],$$

$$c_i=\frac{1}{h^2}$$

as a result, from the primary problem (11)-(16), taking into account (20), we obtain the following spectral problem:

$$a_i\psi_0^i-b_i\psi_0^i-c_i\psi_0^{i-1}=\lambda\psi_0^i, \tag{21}$$

which, in principle, can allow one to find the corresponding eigenfunction, and in general, check the numerical results of [4].

Numerical analysis of Eq.(21) will be implemented in one of our forthcoming publications.

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