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THE ELECTROMAGNETIC FIELD AND RADIATION REACTION FORCE FOR A POINT CHARGED PARTICLE WITH A MAGNETIC MOMENT

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Abstract:
We have obtained an expression for retarded potential of electromagnetic field for arbitrary moving point charged particle having a magnetic moment (corresponding terms are added to Lienard-Wiechert potential). The radiation reaction force which corresponds to radiation part of this field is calculated as well.

Keywords: *Magnetic moment; retarded potential; radiation reaction force; Abraham-Lorentz-Dirac force.*

I. INTRODUCTION

It is well-known that classical electrodynamics contains some problems and contradictions, such as: a) for the hyperbolic motion the radiation reaction (RR) 4-force acting on accelerated charged particle is zero. The problem occurs especially for the time-like component of the RR force while power of radiated energy is not zero; b) self-acceleration phenomenon, when the speed of the charged particle increases under the action of RR force, etc. (see [1]-[5] and references cited there). As it is noted in [5, §75], the problem of a self-acceleration phenomenon has to be considered in connection with the known problems of so called electromagnetic mass (see e.g. [6] v.2]). In the articles [4] we have studied the next problem: the electromagnetic field (EMF) radiated by linearly moving accelerated charge at certain time t_0, after time interval Δt reaches the sphere with radius $R = c\Delta t$ and creates an energy-momentum flux through this sphere. The problem is that the flux decreases as Δt (and R) increases. In order to clarify this problem most authors (see, e.g., [5]) consider such part of EMF that gives flux which does not change as sphere radius increases and call this part the radiation field. Some authors (e.g., [2]) combine the residual field with charged particle to create a "complex" and assume that this two objects somehow exchange an energy.

Rather often (a lot of examples can be cited), problems and contradictions in theoretical Physics occur when and due to incorrect and/or incomplete theoretical model is used to describe a phenomenon. In this context we would like to note that there are no charged fundamental particles (quarks, leptons and gage bosons) without magnetic moment (we restrict ourselves by consideration of only fundamental particles known in the Standard model and found experimentally). So, we could expect that for charge having a magnetic moment the problems noted above will get an explanation. As it will be shown below, this expectation are not materialize. In spite of this, in our opinion, the expressions for retarded potentials and RR force for a charge particle having a magnetic moment are interesting.

In order to calculate the potential of the field created by magnetic moment of arbitrary moving point particle, we start with expression given by J. Frenkel [7]

$$\varphi_k(t,\vec{x}) = \frac{-4}{2\pi i}\oint_{\Gamma} \frac{\mu_{kj} X^j}{X^4} d\tau. \tag{1.1}$$

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Here and below $X^i = x^i - x'^i$ $(i = 0,1,2,3)$; $x^i = (ct, x^\alpha)$ stands for the observation point's 4-coordinates and $x'^i = (ct', x'^\alpha)$ – for the ones of the particle location point $(\alpha = 1,2,3)$; $c = 3\cdot 10^8\ m/s$ is speed of light in vacuum; τ denotes a proper time of the particle; μ_{kj} (see the formula (3.2) below) characterizes the magnetic moment of the particle; Γ is any closed counter on complex plane surrounding the origin. In (1.1) and below we omit the sign of summarization in expressions where indices are repeated.

In the Sec. II, to check the correctness of this approach we consider the similar expression (2.1) for point charged particle and get the correct expression for corresponding potential – Lienard-Wiechert potential (2.2) and correct expression for RR force associated with radiation field in co-moving inertial reference frame (the Larmor's formula). Having written it in covariant form and taking into account the known condition $F^i u_i = 0$, we obtain the expression for the Abraham-Lorentz-Dirac (ALD) force. In the Sec. III the potential of EMF of point particle having magnetic moment is obtained using the formula (1.1). In the Sec. IV we repeat the Sec. II scenario for point particle having magnetic moment; in the Sec. V the same program is provided for charged point particle having magnetic moment. Some special regimes of motion are considered as well.

In the units we use below $c = 1$; the metric tensor is chosen as $g^{ik} = g_{ik} = \mathrm{diag}(1,-1,-1,-1)$, $g^i_j = \delta^i_j = \mathrm{diag}(1,1,1,1)$. Then

$$X^2 = g_{ik}X^iX^k = (x_k - x'_k)(x^k - x'^k) = (t-t')^2 - \vec{R}^2. \tag{1.2}$$

Here

$$\vec{R} = \vec{x} - \vec{x}', \qquad \vec{R}^2 = R^2 = (x^\alpha - x'^\alpha)(x^\alpha - x'^\alpha). \tag{1.3}$$

Brackets $\langle\ \rangle$ stands for a scalar product of 3- and 4-vectors; $\dot{f}(\tau) = df(\tau)/d\tau$ for any function $f(\tau)$.

II. THE RADIATION PART OF THE ELECTROMAGNETIC FIELD OF A POINT CHARGED PARTICLE AND CORRESPONGING RR FORCE

a) J. Frenkel [7] expressed the potential of the charged particle by integral on complex plane (we have changed the metric in the formula (24) of the article [7]):

$$\phi_k(t,\vec{x}) = \frac{-2e}{2\pi i}\oint_\Gamma \frac{dx'_k}{X^2} = \frac{-2e}{2\pi i}\oint_\Gamma \frac{dx'_k}{d\tau}\frac{1}{X^2}d\tau, \tag{2.1}$$

There are two real poles $R = \pm X^0 = \pm(t-t')$, $t > t'$, inside the closed curve Γ. Below we consider only retarded potential. For this case, according to the residue theorem, one obtains the well-known Lienard-Wiechert potential:

$$\phi_k(t,\vec{x}) = -2e\left\{u_k\left(\frac{dX^2}{d\tau}\right)^{-1}\right\}\Bigg|_{R=+X^0} = e\frac{u_k}{\langle Xu\rangle}\Bigg|_{R=+X^0}, \tag{2.2}$$

where

$$u_k = u_k(t') = g_{kl}\,dx'^l/d\tau = -g_{kl}\,dX^l/d\tau.$$

b) Using $\phi_k(t,\vec{x})$ one can calculate the EMF and associated with it 4-momentum passing through the R radius sphere during the time interval $d\tau$ (the charge was in the center of the sphere at time t' when the EMF were radiated). Let us consider only such part of the field that is proportional to R^{-1} (radiation field, see Sec. I; below we use the notation $\hat{\vec{r}} = \vec{R}/R$, $\vec{\upsilon} = d\vec{x}'/dt'$, $\vec{a} = d\vec{\upsilon}/dt'$, $\vec{a}' = d\vec{a}/dt'$).

$$\vec{E}_{eRad.}(t,\vec{x}) = \frac{e\hat{\vec{r}}\times\left[(\hat{\vec{r}} - \vec{\upsilon})\times\vec{a}\right]}{R\left(1 - \langle\hat{\vec{r}}\vec{\upsilon}\rangle\right)^3}, \tag{2.3}$$

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$$\vec{H}_{eRad.}(t,\vec{x}) = (\hat{\vec{r}} \times \vec{E}_{eRad.}). \tag{2.4}$$

where $\vec{x}'$, $\vec{\upsilon}$ and $\vec{a}$ are taken at the retarded time $t' = t - R$.

The 4-momentum which is associated with the field passing through the R radius sphere during the time interval $d\tau$ does not depend on R, so this 4-momentum is assumed to be the radiated 4-momentum.

Let us consider co-moving inertial reference frame – the proper frame (PF) where charge is at rest at the time t'. In this reference frame one has

$$\vec{E}^{PF}_{eRad.}(t,\vec{x}) = \frac{e}{R}\left(\hat{\vec{r}}\left\langle\hat{\vec{r}}\vec{a}\right\rangle - \vec{a}\right)\Big|^{PF}, \qquad \vec{H}^{PF}_{eRad.}(t,\vec{x}) = \frac{e}{R}(\vec{a}\times\hat{\vec{r}})\Big|^{PF}. \tag{2.5}$$

Note, that the relations

$$E^{PF\ 2}_{eRad.} = H^{PF\ 2}_{eRad.}, \qquad \left\langle \vec{E}^{PF}_{eRad.}\vec{H}^{PF}_{eRad.}\right\rangle = 0, \tag{2.6}$$

similar to ones for the plane wave are fulfilled.

Densities of the energy and of the momentum corresponding to (2.5) can be expressed as follows:

$$\frac{E^{PF\ 2}_{eRad.} + H^{PF\ 2}_{eRad.}}{8\pi} = \frac{e^2}{4\pi R^2}\left(a^2 - \left\langle\hat{\vec{r}}\vec{a}\right\rangle^2\right)\Bigg|^{PF}, \tag{2.7}$$

$$\frac{\vec{E}^{PF}_{eRad.} \times \vec{H}^{PF}_{eRad.}}{4\pi} = \frac{e^2}{4\pi R^2}\hat{\vec{r}}\left(a^2 - \left\langle\hat{\vec{r}}\vec{a}\right\rangle^2\right)\Bigg|^{PF}. \tag{2.8}$$

Calculating the energy and the momentum which flow through the R radius sphere during the proper time interval $d\tau = (1-\upsilon^2)^{1/2}dt' = \gamma^{-1}dt'$ one gets (in spherical coordinates)

$$dW^{PF}_{eRad.} = d\tau\int_0^{2\pi} d\varphi\int_0^{\pi} R^2\sin\theta d\theta\frac{e^2}{4\pi R^2}\left(a^2 - \left\langle\hat{\vec{r}}\vec{a}\right\rangle^2\right)\Bigg|^{PF} = (2/3)\,e^2a^2\Big|^{PF}d\tau, \tag{2.9}$$

$$d\vec{P}^{PF}_{eRad.} = d\tau\int_0^{2\pi} d\varphi\int_0^{\pi} R^2\sin\theta d\theta\,\frac{e^2\hat{\vec{r}}}{4\pi R^2}\left(a^2 - \left\langle\hat{\vec{r}}\vec{a}\right\rangle^2\right)\Bigg|^{PF} = 0. \tag{2.10}$$

Relation (2.9) is called the Larmor formula. According to the energy-momentum conservation law the formulas (2.9) and (2.10) mean that a 4-force acting on a charge when corresponding fields were radiated (at the retarded time $t' = t - R$) is:

$$F^{i\,PF}_{eRad.} = -(\dot{W}^{PF}_{eRad.}, \dot{\vec{P}}^{PF}_{eRad.}) = \left(-(2/3)e^2a^2, \vec{0}\right)\Big|^{PF}. \tag{2.11}$$

Presenting (2.11) in covariant form (see App. A) one gets:

$$F^i_{eRad} = (2/3)e^2\omega^2u^i, \tag{2.12}$$

where $\omega^2 = \omega^k\omega_k = \left\langle\dot{u}\dot{u}\right\rangle$, $\omega^k = \dot{u}^k = d/d\tau(\gamma, \gamma\vec{\upsilon})$.

Taking into consideration that

$\left\langle uu\right\rangle = 1 \quad \Rightarrow \quad \left\langle u\omega\right\rangle = (1/2)\,d/d\tau\left\langle uu\right\rangle = 0 \quad \Rightarrow$

$$\Rightarrow \qquad \omega^2 = -u^k d\omega_k/d\tau = -u^k d^2u_k/d\tau^2, \tag{2.13}$$

one can rewrite (2.12) as

$$F^i_{eRad} = -(2/3)\,e^2u^iu^kd^2u_k/d\tau^2. \tag{2.14}$$

The expression (2.14) does not satisfy the condition

$$F^iu_i = 0 \tag{2.15}$$

(in 3-dimension form this condition is equivalent to obvious equality $-dW/dt = \left\langle\vec{F}\vec{\upsilon}\right\rangle$). The reason is we have omitted the non-radiation parts of EMF. Correct calculation of EMF at small distances from

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a charge [8] shows that one has to assume for the RR force, which is associated with non-radiation part of EMF the following expression

$$F^i_{enonRad.} = (2/3)e^2 d^2u^i/d\tau^2. \quad (2.16)$$

(the Schott term). Then the force $F^i_{eRad} + F^i_{enonRad.}$ will satisfy general condition (2.15).

Finally, one obtains the well-known ALD force as follows:

$$F^i_e = F^i_{eRad} + F^i_{enonRad.} = (2/3)e^2 (g^{ik} - u^i u^k) d^2u_k/d\tau^2. \quad (2.17)$$

Note, that to obtain the Schott term most authors (see, e.g. [9]) obtain the Larmor formula (2.9) ([9], (19.19)) considering slowly ($\upsilon \ll c$) moving particle and using approximate expressions for EMF; hence, they assumed that obvious relation $-dW/dt = \langle \vec{F}\,\vec{\upsilon} \rangle$ is satisfied not instantly, but only when it is averaged over the time ([9], p. 346). In contrast, we have obtained the Larmore formula in PF using exact expressions for radiated EMF and then have obtained the Schott term assuming that the condition $-dW/dt = \langle \vec{F}\,\vec{\upsilon} \rangle$ is satisfied at any time moment.

It should be noted that no other terms having a force dimension except terms given in the formula (2.17) can be constructed (using u^i and its time-derivatives).

III. POTENTIAL OF THE ELECTROMAGNETIC FIELD OF THE POINT PARTICLE WITH A MAGNETIC MOMENT

a) In order to calculate the potential of an EMF of the point particle having the magnetic moment J. Frenkel has used the expression similar to (2.1) ([7], the formula (25a)):

$$\varphi_M{}^k(t,\vec{x}) = \frac{-4}{2\pi i} \oint_\Gamma \frac{\mu^{kl}(\tau) X_l}{X^4} d\tau, \quad (3.1)$$

where $\mu^{kl}(\tau)$ is defined as

$$\mu^{kl}(\tau) = \begin{pmatrix} 0 & -d_x & -d_y & -d_z \\ d_x & 0 & -m_z(\vec{\upsilon}) & m_y(\vec{\upsilon}) \\ d_y & m_z(\vec{\upsilon}) & 0 & -m_x(\vec{\upsilon}) \\ d_z & -m_y(\vec{\upsilon}) & m_x(\vec{\upsilon}) & 0 \end{pmatrix}. \quad (3.2)$$

Here $\vec{d}$ is similar to particle's electric dipole moment [7]; $\vec{d}$ and $\vec{m}(\vec{\upsilon})$ have to be find as Lorentz transformation of the following skew-symmetric 4-tensor:

$$\mu^{(PF)kl} = \mu_{kl}{}^{(PF)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -m_z & m_y \\ 0 & m_z & 0 & -m_x \\ 0 & -m_y & m_x & 0 \end{pmatrix}. \quad (3.3)$$

So, in the reference frame where particle moves with the velocity $\vec{\upsilon}$ one has

$$\vec{d} = \gamma \vec{\upsilon} \times \vec{m}(\vec{\upsilon}) = \vec{u} \times \vec{m}(\vec{\upsilon}), \quad (3.4)$$

$$\vec{m}(\vec{\upsilon}) = \gamma \vec{m} - (\gamma - 1)\vec{\upsilon} \langle \vec{\upsilon}\,\vec{m} \rangle \upsilon^{-2}. \quad (3.5)$$

The only nonzero invariant of the skew-symmetric tensor $\mu_{kl}(\tau)$ – the expression

$$\mu^2 = \mu_{kl}\mu^{kl} = 2\left[\vec{m}^2(\vec{\upsilon}) - \vec{d}^{\,2}\right] = 2\left[\vec{m}^2(\vec{\upsilon}) - \upsilon_\perp{}^2 \vec{m}^2(\vec{\upsilon})\right] = 2\vec{m}^2$$

gives

$$\left|\vec{m}(\vec{\upsilon})\right| = (1 - \upsilon_\perp{}^2)^{-1/2} \left|\vec{m}\right|,$$

where $\vec{\upsilon}_\perp$ is perpendicular to $\vec{m}(\vec{\upsilon})$.

Note, that

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$$\mu_{kl} u^l = 0, \tag{3.6}$$

and therefore

$$\dot{\mu}_{kl} u^l + \mu_{kl} \omega^l = 0, \quad \ddot{\mu}_{kl} u^l + 2\dot{\mu}_{kl} \omega^l + \mu_{kl} \dot{\omega}^l = 0, \tag{3.7}$$

and so on.

b) It is well-known ([10] p. 84, (3)) that if a function $F(z)$ has a second-order pole then one has

$$I = \frac{1}{2\pi i} \oint_\Gamma F(z) dz = \lim_{z \to z_0} \frac{d}{dz} \left\{ (z - z_0)^2 F(z) \right\}, \quad (z_0 \notin \Gamma). \tag{3.8}$$

In a special case

$$I = \frac{1}{2\pi i} \oint_\Gamma \frac{f(z)}{\psi(z)} dz \tag{3.9}$$

where $f(z)$ has no poles inside the closed curve Γ while

$$\psi(z) = \frac{\psi''(z_0)}{2!}(z - z_0)^2 + \frac{\psi'''(z_0)}{3!}(z - z_0)^3 + O\left((z - z_0)^4\right), \tag{3.10}$$

the formula (3.9) gives

$$I = \lim_{z \to z_0} \frac{d}{dz} \left\{ \frac{f(z)}{\psi''(z_0)/2! + (z - z_0)\psi'''(z_0)/3! + O((z - z_0)^2)} \right\} =$$

$$= \frac{2f'(z_0)}{\psi''(z_0)} - \frac{2f(z_0)}{3\left(\psi''(z_0)\right)^2} \psi'''(z_0). \tag{3.11}$$

To use the last formula for calculating the potential (3.1) one has to consider the derivatives of X^4 (see (1.2)):

$$\partial X^4 / \partial \tau \Big|_{R = X^0} = -4X^2 \langle Xu \rangle \Big|_{R = X^0} = 0, \tag{3.12}$$

$$\partial^2 X^4 / \partial \tau^2 \Big|_{R = X^0} = 4\left[2\langle Xu \rangle^2 - X^2 \left(\langle X\omega \rangle - 1 \right) \right] \Big|_{R = X^0} = 8\langle Xu \rangle^2 \Big|_{R = X^0}, \tag{3.13}$$

$$\partial^3 X^4 / \partial \tau^3 \Big|_{R = X^0} = 4\left[6\langle Xu \rangle \left(\langle X\omega \rangle - 1 \right) - X^2 X^i \partial \omega_i / \partial \tau \right] \Big|_{R = X^0} = 24\langle Xu \rangle \left(\langle X\omega \rangle - 1 \right) \Big|_{R = X^0}. \tag{3.14}$$

So, according to formulas (3.1), (3.11)-(3.14) and taking into account the formula (3.6), one obtains

$$\varphi_M^k(t, \vec{x}) = \left\{ \frac{X_l}{\langle Xu \rangle^2} \left[\dot{\mu}^{lk} + \mu^{lk} \frac{1 - \langle X\omega \rangle}{\langle Xu \rangle} \right] \right\} \Bigg|_{X^0 = R}, \tag{3.15}$$

where μ^{lk}, u^k, ω^k are taken at the retarded time $t' = t - R$, $\vec{R} = \vec{x} - \vec{x}'(t')$.

Below for sake of brevity we use the following notations:

$$\rho = \langle Xu \rangle \Big|_{R = X^0} = \gamma\left(R - \langle \vec{R}\vec{\upsilon} \rangle \right), \tag{3.16}$$

$$\xi^l = \frac{X^l}{\langle Xu \rangle} \Bigg|_{R = X^0} = \rho^{-1}\left(R, \vec{R} \right) = \gamma^{-1}\left(1 - \langle \hat{\vec{r}}\vec{\upsilon} \rangle \right)^{-1}\left(1, \hat{\vec{r}} \right). \tag{3.17}$$

Taking into account that [5, § 63]

$$\partial_k \tau \Big|_{R = X^0} = g_{lk} X^l \langle Xu \rangle^{-1} \Big|_{R = X^0} = \xi_k, \tag{3.18}$$

it is easy to check that the following relations are valid:

$$\partial_k X^i \Big|_{R = X^0} = \delta_k^i + \xi_k \, \partial X^i / \partial \tau \Big|_{R = X^0} = \delta_k^i - \xi_k u^i, \tag{3.19}$$

$$\partial_k \rho = u_k + \xi_k \left(\langle X\omega \rangle \Big|_{R = X^0} - 1 \right) = u_k + \xi_k \left(\rho \langle \xi\omega \rangle - 1 \right), \tag{3.20}$$

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$$\partial_k \xi^l = \rho^{-1}(\delta_k^l - \xi_k u^l - \xi^l u_k + \xi^l \xi_k) - \xi^l \xi_k \langle \xi\omega \rangle, \tag{3.21}$$

$$\partial_n u^k = \xi_n \omega^k,\ \xi^2 = \xi_k \xi^k = 0,\ \ \xi_k u^k = 1,\ \ \partial_k \xi^k = 2\rho^{-1}. \tag{3.22}$$

Here and below all ρ, ξ^k, u^k are taken at the retarded time $t' = t - R,$ unless otherwise is indicated.

Using (3.6) and formulas (3.16)-(3.21) the potential (3.15) can be rewritten as follows:

$$\varphi_M^k(t,\vec{x}) = \partial_l(\mu^{lk}/\rho). \tag{3.23}$$

Note, that the formula (3.15) differs from one obtained by Frenkel [6, (26)]. In the notations used above the Frenkel's result gets the form:

$$\varphi_{M\,(Fr)}^k(t,\vec{x}) = \frac{\xi_l}{\rho}\left[\dot{\mu}^{lk} + \mu^{lk}\left(\gamma\frac{1+\langle \hat{\vec{r}}\vec{\upsilon}\rangle}{R} - \gamma^3\langle \vec{\upsilon}\vec{a}\rangle\right)\right], \tag{3.24}$$

while the potential (3.15) we have obtained above in the same notations can be presented as:

$$\varphi_M^k(t,\vec{x}) = \frac{\xi_l}{\rho}\left[\dot{\mu}^{lk} + \mu^{lk}\left(\frac{1-\gamma^2\langle \vec{R}\vec{a}\rangle}{\rho} - \gamma^3\langle \vec{\upsilon}\vec{a}\rangle\right)\right]. \tag{3.25}$$

It is obvious that potential (3.1) must satisfy a condition

$$\partial_k \varphi_M^k(t,\vec{x}) = 0 \tag{3.26}$$

(Lorentz gauge) as far the integration in (3.1) is done over a closed curve. Using (3.16)-(3.22) it is easy to show that (3.15) satisfies this condition while the potential (3.24) does not. The contradiction is caused by incorrect transformations in [7] when the theory of residuals was used. Namely, in [7] the expansion (3.10) was hold the first addend only (other terms of the expansion were omitted) while, according to the general formula (3.8) the second term of (3.10) gives significant contribution in the final result (3.15) (or (3.23)). Therefore the formula (3.24) ((26) in [7]) does not correspond to the formula (3.1) ((25a) of [7]).

c) For motionless particle the formula (3.15) (according to App. A and App. B) turns into

$$\varphi^0 = 0, \qquad \vec{\varphi} = \frac{\dot{\vec{m}}\times\hat{\vec{r}}}{R} + \frac{\vec{m}\times\hat{\vec{r}}}{R^2}; \tag{3.27}$$

if $\vec{m} = \text{const}$ then

$$\varphi^0 = 0, \qquad \vec{\varphi} = \frac{\vec{m}\times\hat{\vec{r}}}{R^2}, \tag{3.27'}$$

Below everywhere we shall assume that $\vec{m} = \text{const}$.

Note, that (3.27') coincides with 4-potential of an elementary circular current.

d) As far the potential (3.15) (and hence (3.23)) is in Lorentz gauge the Maxwell equation gets the form

$$\partial_n F^{nk}(t,\vec{x}) = \partial_n \partial^n \varphi^k(t,\vec{x}) = 4\pi j^k(t,\vec{x}).$$

So, for the potential (3.23) we have

$$\partial_n \partial^n \partial_l(\mu^{lk}\rho^{-1}) = \partial_l \partial_n \partial^n(\mu^{lk}\rho^{-1}) = 4\pi j^k(t,\vec{x}). \tag{3.28}$$

Taking into consideration the Leibnitz formula one obtains

$$j^k(t,\vec{x}) = (4\pi)^{-1}\partial_l \partial^n \partial_n(\mu^{lk}\rho^{-1}) =$$
$$= (4\pi)^{-1}\partial_l(\mu^{lk}\partial^n\partial_n\rho^{-1}) + (4\pi)^{-1}\left(2(\partial^n\rho^{-1})(\partial_n\mu^{lk}) + \rho^{-1}\partial^n\partial_n\mu^{lk}\right). \tag{3.29}$$

In order to calculate $\partial^n\partial_n\rho^{-1}$ one can use the known result [5, § 63]

$$\partial^n\partial_n(u_k\rho^{-1}) = 4\pi\gamma^{-1}(t)u_k(t)\delta(\vec{R}).$$

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It should be emphasized that in the last formula due to $R=0$ time is not retarded, so we have dependence on the observer's time: $u_k(t)$, $\gamma^{-1}(t)$, $\mu^{lk}(t)$. Then, taking into account (2.13) and (3.22) one obtains

$$\partial^n\partial_n\rho^{-1} = \partial^n\partial_n(u^k u_k\rho^{-1}) = u^k\partial^n\partial_n(u_k\rho^{-1}) + 2(\partial^n u^k)\partial_n(u_k\rho^{-1}) + u_k\rho^{-1}\partial^n\partial_n u^k =$$
$$= u^k(t)\left(4\pi\gamma^{-1}(t)u_k(t)\,\delta(\vec{R})\right) = 4\pi\gamma^{-1}(t)\,\delta(\vec{R}).$$

Inserting this result and (3.20)-(3.22) in the formula (3.29), after obvious simplifications one finds

$$j^k(t,\vec{x}) = \partial_l\left(\mu^{lk}\gamma^{-1}\delta(\vec{R})\right) + (4\pi)^{-1}\partial_l\left(\rho^{-1}\partial^n(\dot{\mu}^{lk}\xi_n) - 2\rho^{-2}\dot{\mu}^{lk}\right) =$$
$$= \partial_l\left(\mu^{lk}\gamma^{-1}\delta(\vec{R})\right) + \partial_l(4\pi\rho)^{-1}\left(\ddot{\mu}^{lk}\xi^n\xi_n + \dot{\mu}^{lk}\partial^n\xi_n - 2\rho^{-1}\dot{\mu}^{lk}\right).$$

Finally, according to (3.22) we find

$$j^k(t,\vec{x}) = \partial_l\left(\mu^{lk}(t)\gamma^{-1}(t)\delta(\vec{R})\right). \tag{3.30}$$

So, taking into account that $\mu^{00}=0$ (see (3.2)), one finds components of magnetic moment having particle's current:

$$j^0(t,\vec{x}) = \partial_\alpha\left[\mu^{\alpha 0}(t)\gamma^{-1}(t)\delta\left(\vec{x}-\vec{x}'(t)\right)\right] = \mu^{\alpha 0}(t)\gamma^{-1}(t)\,\partial_\alpha\delta\left(\vec{x}-\vec{x}'(t)\right), \tag{3.31}$$

$$j^\alpha(t,\vec{x}) = \delta\left(\vec{x}-\vec{x}'(t)\right)\frac{\partial}{\partial t}\frac{\mu^{0\alpha}(t)}{\gamma(t)} + \frac{\mu^{\beta\alpha}(t)-\mu^{0\alpha}(t)\upsilon^\beta(t)}{\gamma(t)}\partial_\beta\,\delta\left(\vec{x}-\vec{x}'(t)\right). \tag{3.32}$$

It is easy to show that the appropriate magnetic charge does not exist: calculating the space integral of the $j^0(t,\vec{x})$ according to the Gauss theorem we obtain

$$\int_V j^0(t,\vec{x})\,dV = \int_V \partial_\alpha\left[\mu^{\alpha 0}(t)\gamma^{-1}(t)\delta\left(\vec{x}-\vec{x}'(t)\right)\right]dV = \oint_{\partial V}\mu^{\alpha 0}(t)\gamma^{-1}(t)\delta\left(\vec{x}-\vec{x}'(t)\right)dS^\alpha = 0. \tag{3.33}$$

IV. THE RADIATION PART OF EMF OF THE POINT PARTICLE HAVING A MAGNETIC MOMENT, AND THE RR FORCE

Using (3.18)-(3.22) one can calculate EMF tensor corresponding to the potential (3.15) as follows:

$$F_{ik}(t,\vec{x}) = \partial_i\varphi_k(t,\vec{x}) - \partial_k\varphi_i(t,\vec{x}) =$$
$$= \rho^{-3}\left(\rho\langle\xi\omega\rangle - 1\right)\left\{3\left[u_i + \left(\rho\langle\xi\omega\rangle-1\right)\xi_i\right]\xi^n\mu_{nk} - \mu_{ik}\right\} +$$
$$+\rho^{-2}\left\{\dot{\mu}_{ik} - \left[3\xi_i\xi^n\left(\rho\langle\xi\omega\rangle-1\right) + 2u_i\xi^n + \xi_i u^n\right]\dot{\mu}_{nk} - \omega_i\mu_{nk}\xi^n\right\} +$$
$$+\rho^{-1}\xi_i\xi^n\left[\ddot{\mu}_{nk} - \langle\xi\dot{\omega}\rangle\mu_{nk}\right] - [\text{terms with } i \rightleftarrows k]. \tag{4.1}$$

Let us consider a part of the tensor F_{ik} that is proportional to R^{-1} (the radiation part of the field):

$$F_{ik\,Rad}(t,\vec{x}) =$$
$$= \xi^n\rho^{-1}\left[(\xi_i\mu_{nk} - \xi_k\mu_{ni})\left(3\langle\xi\omega\rangle^2 - \langle\xi\dot{\omega}\rangle\right) - 3(\xi_i\dot{\mu}_{nk} - \xi_k\dot{\mu}_{ni})\langle\xi\omega\rangle + (\xi_i\ddot{\mu}_{nk} - \xi_k\ddot{\mu}_{ni})\right]. \tag{4.2}$$

So, in the co-moving inertial reference frame for $\vec{E}^{PF}_{M\,Rad.}$ and $\vec{H}^{PF}_{M\,Rad.}$ we obtain the following expressions:

$$\vec{E}^{PF}_{M\,Rad.}(t,\vec{x}) = R^{-1}\left[\hat{\vec{r}}\left(3\langle\hat{\vec{r}}\vec{a}\rangle\langle\vec{m}\left(\hat{\vec{r}}\times\vec{a}\right)\rangle + \langle\vec{m}\left(\hat{\vec{r}}\times\vec{a}'\right)\rangle\right) +\right.$$
$$\left.\left. +3\langle\hat{\vec{r}}\,\vec{a}\rangle\left(\vec{m}\times\vec{a}\right) + \left(\vec{m}\times\vec{a}'\right) - \langle\vec{m}\,\vec{a}\rangle\left(\hat{\vec{r}}\times\vec{a}\right) - \left(\vec{m}\times\hat{\vec{r}}\right)\left(3\langle\hat{\vec{r}}\vec{a}\rangle^2 + \langle\hat{\vec{r}}\vec{a}'\rangle\right)\right]\right|^{PF}, \tag{4.3}$$

$$\vec{H}^{PF}_{M\,Rad.}(t,\vec{x}) = R^{-1}\left\{\hat{\vec{r}}\left[\langle\vec{m}\hat{\vec{r}}\rangle\left(3\langle\hat{\vec{r}}\vec{a}\rangle^2 + \langle\hat{\vec{r}}\vec{a}'\rangle\right) - \langle\vec{a}\,\vec{m}\rangle\langle\hat{\vec{r}}\vec{a}\rangle\right] +\right.$$
$$\left.\left. +\vec{a}\left(-3\langle\vec{m}\hat{\vec{r}}\rangle\langle\hat{\vec{r}}\vec{a}\rangle + \langle\vec{m}\,\vec{a}\rangle\right) + \vec{m}a^2 - \vec{a}'\langle\vec{m}\hat{\vec{r}}\rangle\right\}\right|^{PF}. \tag{4.4}$$

To obtain these formulas we have used results shown in App. *A*; $\vec{a}$ and $\vec{a}' = d\vec{a}/dt'$ are taken at retarded time $t' = t - R$.

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Note, that similarly to (2.6) the following relations are fulfilled:

$$E_{M\,Rad.}^{PF\;2} = H_{M\,Rad.}^{PF\;2}, \qquad \langle \vec{E}_{M\,Rad.}^{PF} \vec{H}_{M\,Rad.}^{PF} \rangle = 0. \tag{4.5}$$

Repeating for these fields calculations performed in Sec. II, similarly to (2.9) and (2.10), one gets:

$$dW_{M\,Rad.}^{PF} = d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} R^2 \sin\theta d\theta \left(E_{MRad.}^{PF\;2} + H_{MRad.}^{PF\;2} \right)/8\pi =$$
$$= (2/105) d\tau \left\{ 2a^2 \left(\langle \vec{m}\vec{a} \rangle^2 + 9\vec{m}^2 a^2 \right) - 7\left(\langle \vec{a}'\vec{m} \rangle^2 - 2a'^2 \vec{m}^2 \right) \right\} \Big|^{PF}, \tag{4.6}$$

$$d\vec{P}_{M\,Rad.}^{PF} = d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} R^2 \sin\theta d\theta \left(\vec{E}_{MRad.}^{PF} \times \vec{H}_{M\,Rad.}^{PF} \right)/4\pi =$$
$$= (2/105) d\tau \left\{ 2\vec{m}\left(\langle \vec{a}\vec{m} \rangle \langle \vec{a}\vec{a}' \rangle - 3a^2 \langle \vec{a}'\vec{m} \rangle \right) + \vec{a}'\left(\langle \vec{a}\vec{m} \rangle^2 - 3\vec{m}^2 a^2 \right) - 5\vec{a}\left(\langle \vec{a}\vec{m} \rangle \langle \vec{a}'\vec{m} \rangle - 3\vec{m}^2 \langle \vec{a}'\vec{a} \rangle \right) \right\} \Big|^{PF}. \tag{4.7}$$

Similarly to (2.11) we get

$$F_{M\,Rad.}^{0\,PF} = (2/105)\left[7\left(\langle \vec{a}'\vec{m} \rangle^2 - 2a'^2 \vec{m}^2 \right) - 3a^2 \left(\langle \vec{a}\vec{m} \rangle^2 - a^2 \vec{m}^2 \right) \right] \Big|^{PF},$$
$$\vec{F}_{M\,Rad}^{PF} = (2/105)\left[2\vec{m}\left(\langle \vec{a}\vec{m} \rangle \langle \vec{a}\vec{a}' \rangle - 3a^2 \langle \vec{a}'\vec{m} \rangle \right) - \vec{a}'\left(\langle \vec{a}\vec{m} \rangle^2 - 3a^2 \vec{m}^2 \right) - \right.$$
$$\left. -5\left(\langle \vec{a}\vec{m} \rangle \langle \vec{a}'\vec{m} \rangle - 3\vec{m}^2 \langle \vec{a}\vec{a}' \rangle \right) \right] \Big|^{PF}. \tag{4.8}$$

Rewriting (4.8) in covariant form (see App. D) one gets:

$$F_{M\,Rad}^{i} = (1/105)\left\{ -2\left[5\omega_k \mu^{km} \ddot{\mu}_{mn} u^n + 6\mu^2 \langle \omega\dot{\omega} \rangle \right] \omega^i + \right.$$
$$+ \left[27\mu^2(\omega^4 - 7\dot{\omega}^2) + 14(\mu_{nk}\dot{\omega}^k)^2 + 4(\mu_{nk}\omega^k)^2 \omega^2 \right] u^i +$$
$$\left. +4\mu^{im}\mu_{mn}\left(\langle \omega\dot{\omega} \rangle \omega^n - 3\omega^2 \dot{\omega}^n \right) + 2\left[4\mu^2 \omega^2 - (\mu_{nk}\omega^k)^2 \right] \dot{\omega}^i \right\}. \tag{4.9}$$

The expression (4.9) does not satisfy the condition (2.15); the reason is the same as above (see Sec. III): we have not taken into account non-radiation parts of the field. If one assumes that Abraham-Lorentz force associated with non-radiation part of the EMF gives the term

$$F_{M\,nonRad.}^{i} = -(2/15)\ddot{\mu}^{im}\mu_{mn}\dot{\omega}^n + (1/15)\mu^2 \dot{\omega}^2 u^i +$$
$$+ (1/105)\left[6(\mu_{nk}\omega^k)^2 + 19\mu^2\omega^2 \right] \dot{\omega}^i, \tag{4.10}$$

then the force $F_{MRad.}^{i} + F_{M\,nonRad.}^{i}$ will satisfy the condition (2.15).

Finally, the expression for the RR force in the case under consideration can be presented as

$$F_M^i = F_{M\,Rad.}^{i} + F_{M\,nonRad.}^{i} =$$
$$= (1/105)\left\{ \left[4\mu^{im}\mu_{mn}(\omega_k\omega^n - 3\omega^2\delta_k^n) + 14(u^i\mu^{ml}\dot{\omega}_l - \ddot{\mu}^{im})\mu_{mk} + \delta_k^i\left(27\mu^2\omega^2 + 4(\mu_{nl}\omega^l)^2 \right) - \right.\right.$$
$$\left.\left. -2\omega^i\left(5\omega_l \mu^{lm}\mu_{mk} + 6\mu^2\omega_k \right) \right] \dot{\omega}^k + u^i\omega^2\left(4(\mu_{nk}\omega^k)^2 + 27\mu^2\omega^2 \right) \right\}. \tag{4.11}$$

The components of this force in proper frame are:

$$F_M^{i\,PF} = (F^0, \vec{F}) \Big|^{PF};$$
$$F_M^{0\,PF} = 0, \quad \vec{F}_M^{PF} = (1/105)\left[10\vec{a}\left(-3\vec{m}^2 \langle \vec{a}\vec{a}' \rangle + \langle \vec{a}\vec{m} \rangle \langle \vec{a}'\vec{m} \rangle \right) - \right.$$
$$\left. -2\vec{a}'(5\langle \vec{a}\vec{m} \rangle^2 - 26a^2\vec{m}^2) + 2\vec{m}\left(5\langle \vec{a}\vec{m} \rangle \langle \vec{a}\vec{a}' \rangle - a^2 \langle \vec{a}'\vec{m} \rangle \right) \right|^{PF}. \tag{4.12}$$

V. RR FORCE ACTING ON THE CHARGED POINT PARTICLE WITH A MAGNETIC MOMENT

Let us consider now the general case - a particle having an electric charge e and a magnetic moment $\vec{m}$ (in proper frame); correspondent fields are

$$\vec{E}(t,\vec{x}) = \vec{E}_e(t,\vec{x}) + \vec{E}_M(t,\vec{x}), \tag{5.1}$$

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$$\vec{H}(t,\vec{x}) = \vec{H}_e(t,\vec{x}) + \vec{H}_M(t,\vec{x}). \qquad (5.2)$$

Radiation part of these fields in proper frame, similarly to (2.6), satisfy relations

$$E_{Rad.}^{PF\,2} = H_{Rad.}^{PF\,2}, \quad \langle \vec{E}_{Rad.}^{PF} \vec{H}_{Rad.}^{PF} \rangle = 0. \qquad (5.3)$$

The density of energy is

$$(E_{Rad.}^{PF\,2} + H_{Rad.}^{PF\,2})/8\pi =$$
$$= (8\pi)^{-1}\Big(E_{eRad.}^{PF\,2} + H_{eRad.}^{PF\,2} + E_{MRad.}^{PF\,2} + H_{MRad.}^{PF\,2} + 2\langle \vec{E}_{eRad.}^{PF} \vec{E}_{MRad.}^{PF} \rangle + 2\langle \vec{H}_{eRad.}^{PF} \vec{H}_{MRad.}^{PF} \rangle\Big). \qquad (5.4)$$

Let us calculate the energy which is flowing through the R radius sphere during time interval $d\tau$. The integrals of the first two terms in the right hand side of (5.4) are given by (2.5), (4.3), (4.4); the last two terms are equal to

$$(4\pi)^{-1}\Big(\langle \vec{E}_{eRad.}^{PF} \vec{E}_{\mathrm{M}Rad.}^{PF} \rangle + \langle \vec{H}_{eRad.}^{PF} \vec{H}_{\mathrm{M}Rad.}^{PF} \rangle\Big) = e(2\pi R^2)^{-1}\langle \vec{m}\hat{\vec{r}} \rangle \langle \hat{\vec{r}}(\vec{a}' \times \vec{a}) \rangle \Big|^{PF}. \qquad (5.5)$$

Hence, the energy flux corresponding to (5.4) is:

$$dW_{Rad.}^{PF} = d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} R^2 \sin\theta d\theta (E_{Rad.}^{PF\,2} + H_{Rad.}^{PF\,2})/(8\pi) =$$
$$= d\tau \Big\{(2e^2/3)a^2 + (2e/3)\langle \vec{m}(\vec{a}' \times \vec{a}) \rangle +$$
$$+ (2/105)\Big[2a^2\big(\langle \vec{a}\vec{m} \rangle^2 + 9a^2\vec{m}^2\big) - 7\big(\langle \vec{a}'\vec{m} \rangle^2 - 2a'^2\vec{m}^2\big)\Big]\Big\}\Big|^{PF}. \qquad (5.6)$$

The density of the field momentum can be presented as

$$(4\pi)^{-1}(\vec{E}_{Rad.}^{PF} \times \vec{H}_{Rad.}^{PF}) =$$
$$= (4\pi)^{-1}\Big\{\vec{E}_{eRad.}^{PF} \times \vec{H}_{eRad.}^{PF} + (\vec{E}_{MRad.}^{PF} \times \vec{H}_{MRad.}^{PF}) +$$
$$+ (\vec{E}_{eRad.}^{PF} \times \vec{H}_{MRad.}^{PF}) + (\vec{E}_{MRad.}^{PF} \times \vec{H}_{eRad.}^{PF})\Big\}. \qquad (5.7)$$

In order to calculate the momentum flowing through the R radius sphere during time interval $d\tau$ one has to use (2.10) and (4.7) for integrals of the first two terms in (5.7); the last two terms in the right hand side of (5.7) are equal to

$$(4\pi)^{-1}(\vec{E}_{eRad.}^{PF} \times \vec{H}_{MRad.}^{PF} + \vec{E}_{MRad.}^{PF} \times \vec{H}_{eRad.}^{PF}) = e(4\pi R^2)^{-1}\hat{\vec{r}}\langle \vec{m}\hat{\vec{r}} \rangle \langle \hat{\vec{r}}(\vec{a}' \times \vec{a}) \rangle \Big|^{PF}, \qquad (5.8)$$

So, the momentum flux corresponding to (5.7) is:

$$d\vec{P}_{Rad.}^{PF} = d\tau(2/105)\Big\{\vec{a}'\big[\langle \vec{a}\vec{m} \rangle^2 - 3a^2\vec{m}^2\big] +$$
$$+ 5\vec{a}\big[3\vec{m}^2\langle \vec{a}\vec{a}' \rangle - \langle \vec{a}'\vec{m} \rangle\langle \vec{a}\vec{m} \rangle\big] + 2\vec{m}\big[\langle \vec{a}\vec{m} \rangle\langle \vec{a}\vec{a}' \rangle - 3a^2\langle \vec{a}'\vec{m} \rangle\big]\Big\}\Big|^{PF}. \qquad (5.9)$$

According to the energy-momentum conservation law this means that the RR force acted on the particle when correspondent fields were radiated (at the retarded time $t' = t - R$) is:

$$F_{Rad}^{i\,PF} = (F^0, \vec{F})\Big|^{PF} = -\Big(\dot{W}_{Rad.}^{PF}, \dot{\vec{P}}_{Rad.}^{PF}\Big); \qquad (5.10)$$

$$F_{Rad.}^{0\,PF} = \Big[-(2e^2/3)a^2 - (2e/3)\langle \vec{m}(\vec{a} \times \vec{a}') \rangle +$$
$$+ (2/105)\big(7\langle \vec{a}'\vec{m} \rangle^2 - 2a'^2\vec{m}^2\big) - 3a^2\big(\langle \vec{a}\vec{m} \rangle^2 + 3a^2\vec{m}^2\big)\Big]\Big|^{PF},$$

$$\vec{F}_{Rad.}^{PF} = -(2/105)\Big\{\vec{a}'[\langle \vec{a}\vec{m} \rangle^2 - 3a^2\vec{m}^2] + 5\vec{a}[3\vec{m}^2\langle \vec{a}'\vec{a} \rangle - \langle \vec{a}\vec{m} \rangle\langle \vec{a}'\vec{m} \rangle] +$$
$$+ 2\vec{m}[\langle \vec{a}\vec{m} \rangle\langle \vec{a}'\vec{a} \rangle - 3a^2\langle \vec{a}'\vec{m} \rangle]\Big\}\Big|^{PF}. \qquad (5.11)$$

Rewriting this force in covariant form one gets (see details in App. C):

$$F_{Rad.}^i = (2e^2/3)\omega^2 u^i + (2e/3)(u^n \dot{\mu}_{nk} \dot{\omega}^k)u^i +$$

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$$+(1/105)\left\{\left[3(9\omega^4-7\dot{\omega}^2)\mu^2+14(\mu^{nm}\dot{\omega}_m)^2+4(\mu^{nm}\omega_m)^2\omega^2\right]u^i-\right.$$
$$-2\left[5\omega_k\mu^{km}\mu_{mn}+6\mu^2\omega_n\right]\dot{\omega}^n\omega^i+2\left[4\mu^2\omega^2-(\mu_{nk}\omega^k)^2\right]\dot{\omega}^i+$$
$$\left.+4\dot{\omega}^n(\omega_n\mu^{im}\mu_{mk}\omega^k-3\omega^2\mu^{im}\mu_{mn})\right\}. \quad (5.12)$$

The expressions (5.12) does not satisfy the condition (2.15). Assuming that a recoil force associated with non-radiation part of the fields $\vec{E}$ and $\vec{H}$ contains the terms

$$F^i_{nonRad.}=(2e^2/3)\dot{\omega}^i-(2e/3)\dot{\mu}^{im}\dot{\omega}_m+(1/105)\left\{7\mu^2\dot{\omega}^2u^i-14\ddot{\mu}^{im}\mu_{mn}\dot{\omega}^n+\left[6(\mu_{nk}\omega^k)^2+19\mu^2\omega^2\right]\dot{\omega}^i\right\},$$

one gets the RR force which satisfies the condition (2.15):

$$F^i=F^i_{Rad}+F^i_{nonRad.}=$$
$$=\dot{\omega}^n\left\{(g^{ik}-u^iu^k)\left[(2/3)e^2g_{kn}+(9/35)\mu^2\omega^2g_{kn}-(2/3)e\dot{\mu}_{kn}+(4/105)(\mu_{lm}\omega^m)^2g_{kn}\right]+\right.$$
$$\left.+(2/105)\left[\left(7u^i\mu^{mk}\dot{\omega}_k-6\mu^{im}\omega^2-7\ddot{\mu}^{im}\right)\mu_{mn}-\omega^i\mu^{km}(5\omega_k\mu_{mn}+6\mu_{km}\omega_n)+2\mu^{im}\mu_{mk}\omega^k\omega_n\right]\right\}. \quad (5.13)$$

So, based on (5.13), one can conclude that for a particle having nonzero magnetic moment the radiation reaction force vanishes if $\dot{\omega}^n=0$. This is well-known "hyperbolic motion" problem [6].

Note, that the result obtained can be presented in some alternative form using identities (2.13), (3.6) and (3.7).

In the co-moving system the expression (5.13) gets the following form (see App. E)

$$F^{i\,PF}=\left(F^0,\vec{F}\right)\Big|^{PF}; \quad (5.14)$$
$$F^{0\,PF}=0,$$
$$\vec{F}^{PF}=(2e^2/3)\vec{a}'+(2e/3)a^2(\vec{a}\times\vec{m})+$$
$$+(1/105)\left\{2\vec{a}'\left[26a^2m^2-5\langle\vec{a}\vec{m}\rangle^2\right]+2\vec{m}\left[5\langle\vec{a}\vec{m}\rangle\langle\vec{a}'\vec{a}\rangle-a^2\langle\vec{a}'\vec{m}\rangle\right]+\right.$$
$$\left.+10\vec{a}\left[\langle\vec{a}\vec{m}\rangle\langle\vec{a}'\vec{m}\rangle-3m^2\langle\vec{a}'\vec{a}\rangle\right]\right\}\Big|^{PF}. \quad (5.15)$$

Example 1. Rectilinear motion. Lorentz transformations of (5.14) and (5.15) give (see details in App. A; here we use the ordinary space-time units);

$$F^i=\left(\frac{2e^2}{3c^3}+\frac{2\vec{m}^2}{7c^7}a^2\gamma^6\right)\left(\frac{da}{dt'}+\frac{3a^2\upsilon}{c^2}\gamma^2\right)\gamma^5\cdot\left(\frac{\upsilon}{c},\hat{\vec{\upsilon}}\right). \quad (5.16)$$

Here and bellow $\hat{\vec{\upsilon}}=\vec{\upsilon}/\upsilon$; a, υ are projections on the direction of motion.

According to (5.16) the magnetic moment of a charged particle cannot be neglected when summands in the first brackets are of the same order:

$$3m^2a^2\gamma^6/7c^4e^2\sim 1, \quad a\gamma^3\sim c^2e/|\vec{m}|.$$

For electron it means that

$$\gamma\simeq 10^{10}a^{-1/3}, \quad \mathcal{E}_e=\gamma m_ec^2\simeq 10^{10}a^{-1/3}m_ec^2.$$

Assuming that $a\sim(1-10^3)\,ms^{-2}$, one gets the estimation of correspondent energy as

$$\mathcal{E}_e\simeq(10^{15}-10^{16})\,eV=(10^3-10^4)\,TeV.$$

Example 2. Uniform circular motion. The corresponding RR force (see App. F) is:

$$F^i=-\left(\frac{2e^2}{3}+\frac{2e|\vec{m}|}{3}\frac{\upsilon^3\gamma^2}{c^3R}+\frac{52\vec{m}^2}{105}\frac{\upsilon^4\gamma^4}{c^4R^2}\right)\frac{\upsilon^3\gamma^5}{c^3R^2}\left(\frac{\upsilon}{c},\hat{\vec{\upsilon}}\right). \quad (5.17)$$

Similarly to the previous example, one can estimate the energy when contribution of the magnetic moment becomes essential. For electron, assuming that $R\simeq 10^3\,m$ one gets:

$$\gamma_e\simeq 7\cdot 10^7 \;\Rightarrow\; \mathcal{E}\sim 3\cdot 10^{13}\,eV\sim 10\;\; TeV.$$

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The estimation obtained indicates that RR force due to nonzero magnetic moment may be essential just for next generation accelerators. Besides, the estimation indicates on a significance of this force in some relativistic astrophysical object. Say, some authors [11 *a*, *b*] have shown that electrons/positrons in Crab nebula have $\gamma_e \simeq 10^6 - 10^9$.

VI. CONCLUSIONS

Expression derived for RR force acting on magnetic moment having particle demonstrates that this force increases rather fast as particle's energy increases. Starting from some threshold, magnetic RR force becomes stronger then electric one and we could expect that it would be taken into account in next generation accelerators in order to choose optimal regime of acceleration. In our opinion, it has to be taken into consideration also in developments of synchrotronic radiation from vicinity of relativistic astrophysical objects (pulsars, magnetars, black holes), as well as in laboratory.

It seems interesting to compare obtained classical results with correspondent calculations, based on quantum theory.

In our opinion, the results obtained may be important in order to reproduce characteristics of cosmic rays having ultrahigh energies.

APPENDIX *A*

Let us express u^i and its derivatives by 3-vectors:

$$u^i = \gamma(1,\vec{\upsilon}), \quad \omega^i = du^i/d\tau = \dot{u}^i = \gamma^4\left(\langle\vec{a}\vec{\upsilon}\rangle, \vec{a}\gamma^{-2} + \vec{\upsilon}\langle\vec{a}\vec{\upsilon}\rangle\right),$$

$$\dot{\omega}^i = \left(\dot{\omega}^0, \dot{\omega}^\alpha\right), \quad \dot{\omega}^0 = \left(a^2 + \langle\vec{a}'\vec{\upsilon}\rangle\right)\gamma^5 + 4\langle\vec{a}\vec{\upsilon}\rangle^2\gamma^7,$$

$$\dot{\vec{\omega}} = \vec{a}'\gamma^3 + \left(3\vec{a}\langle\vec{a}\vec{\upsilon}\rangle + \vec{\upsilon}\langle\vec{a}'\vec{\upsilon}\rangle + \vec{\upsilon}a^2\right)\gamma^5 + 4\vec{\upsilon}\langle\vec{a}\vec{\upsilon}\rangle^2\gamma^7.$$

It is obvious that

$$u^i\omega_i = 0 \qquad \Rightarrow \qquad \omega^2 + u^i\dot{\omega}_i = 0,$$

$$\omega^2 + u^i\dot{\omega}_i = 0, \quad 3\dot{\omega}^i\omega_i + u^i\ddot{\omega}_i = 0, \quad 4\ddot{\omega}^i\omega_i + 3\dot{\omega}^2 + u^i\dddot{\omega}_i = 0, \ldots$$

In proper frame one has:

$$u^{i\,PF} = (1,\vec{0}), \quad \omega^{i\,PF} = \left(0,\vec{a}\right)\Big|^{PF}, \quad d\omega^i/d\tau\Big|^{PF} = \dot{\omega}^i\Big|^{PF} = \left(a^2, \vec{a}'\right)\Big|^{PF},$$

$$a^2\Big|^{PF} = -\omega^2, \quad a'^2\Big|^{PF} = (\omega^4 - \dot{\omega}^2), \quad \langle\vec{a}'\vec{a}\rangle\Big|^{PF} = -\omega^k\dot{\omega}_k = -\langle\omega\dot{\omega}\rangle.$$

APPENDIX *B*

In order to find derivatives of the tensor μ_{ik} (3.2) one has to calculate previously derivatives of $\vec{d}(\vec{\upsilon})$ and $\vec{m}(\vec{\upsilon})$:

$$\dot{\vec{d}} = \vec{\omega}\times\vec{m}, \quad \ddot{\vec{d}}(\tau) = \dot{\vec{\omega}}\times\vec{m};$$

$$\dot{\vec{m}}(\vec{\upsilon}) = \gamma^4\langle\vec{a}\vec{\upsilon}\rangle\left(\vec{m} - \langle\vec{m}\hat{\vec{\upsilon}}\rangle\vec{\upsilon}\right) + (1-\gamma)\gamma\upsilon^{-1}\left(\vec{a}\langle\vec{m}\hat{\vec{\upsilon}}\rangle - 2\langle\vec{a}\hat{\vec{\upsilon}}\rangle\langle\vec{m}\hat{\vec{\upsilon}}\rangle\hat{\vec{\upsilon}} + \langle\vec{m}\vec{a}\rangle\hat{\vec{\upsilon}}\right);$$

$$\begin{aligned}\ddot{\vec{m}}(\vec{\upsilon}) = {} & \gamma^5\Big\{\left[\hat{\vec{\upsilon}}\times(\vec{m}\times\hat{\vec{\upsilon}})\right]\left[a^2 + \langle\vec{a}'\vec{\upsilon}\rangle + 4\langle\vec{a}\vec{\upsilon}\rangle^2\gamma^2\right] + 4\langle\vec{a}\hat{\vec{\upsilon}}\rangle^2\langle\vec{m}\hat{\vec{\upsilon}}\rangle\hat{\vec{\upsilon}} - 2\langle\vec{a}\hat{\vec{\upsilon}}\rangle\left[\langle\vec{m}\hat{\vec{\upsilon}}\rangle\vec{a} + \langle\vec{m}\vec{a}\rangle\hat{\vec{\upsilon}}\right]\Big\} + \\ & + (1-\gamma)\gamma^2\Big\{(4\upsilon^{-2} - 5)\gamma^2\langle\vec{a}\hat{\vec{\upsilon}}\rangle\left((2\hat{\vec{\upsilon}}\langle\vec{a}\hat{\vec{\upsilon}}\rangle - \vec{a})\langle\vec{m}\hat{\vec{\upsilon}}\rangle - \hat{\vec{\upsilon}}\langle\vec{m}\vec{a}\rangle\right) + \\ & + \upsilon^{-2}\left[\vec{a}'\langle\vec{m}\vec{\upsilon}\rangle - 2\langle\vec{m}\hat{\vec{\upsilon}}\rangle\left(a^2\hat{\vec{\upsilon}} + \vec{\upsilon}\langle\vec{a}'\hat{\vec{\upsilon}}\rangle\right) + \vec{\upsilon}\langle\vec{a}'\vec{m}\rangle + 2\vec{a}\langle\vec{m}\vec{a}\rangle\right]\Big\}. \qquad \left(\hat{\vec{\upsilon}} \equiv \vec{\upsilon}/\upsilon\right)\end{aligned}$$

For $\vec{d}$ in the PF we obtain:

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$$\dot{\vec{d}}\Big|^{PF} = \vec{a}^{PF}\times\vec{m}, \qquad \ddot{\vec{d}}\Big|^{PF} = \vec{a}'^{PF}\times\vec{m}.$$

In order to obtain derivatives of $\vec{m}(\vec{\upsilon})$ in PF one has to direct the Ox axis along the instantaneous velocity $\vec{\upsilon}(t')$ and the Oy axis in the plane which contains $\vec{\upsilon}(t')$ and $\vec{a}(t')$. In this case, obviously, one has

$$\dot{\vec{m}}(\vec{\upsilon})\Big|^{PF} = \lim_{\upsilon\to 0}\left\{\gamma^4\left\langle\vec{a}\,\hat{\vec{\upsilon}}\right\rangle\left(\vec{m}-\left\langle\vec{m}\,\hat{\vec{\upsilon}}\right\rangle\vec{\upsilon}\right)+\frac{\gamma(1-\gamma)}{\upsilon}\left[\vec{a}\left\langle\vec{m}\,\hat{\vec{\upsilon}}\right\rangle-2\left\langle\vec{a}\,\hat{\vec{\upsilon}}\right\rangle\left\langle\vec{m}\,\hat{\vec{\upsilon}}\right\rangle\hat{\vec{\upsilon}}+\left\langle\vec{m}\,\vec{a}\right\rangle\hat{\vec{\upsilon}}\right]\right\}=$$

$$=\lim_{\upsilon\to 0}\frac{1-\gamma}{\upsilon}(\alpha\hat{\vec{\upsilon}}+\beta\vec{a}) = 0$$

(here $\alpha=\alpha(t'), \beta=\beta(t')$ denote certain limited real functions).

Similarly,

$$\ddot{m}_\alpha(\vec{\upsilon})\Big|^{PF} = \lim_{\upsilon\to 0}\ddot{m}_\alpha(\vec{\upsilon}) = \left[-a_\alpha\left\langle\vec{m}\,\vec{a}\right\rangle + a^2 m_\alpha\right]\Big|^{PF}, \quad \alpha = 1,2,3.$$

In order to obtain the last formula the obvious equality

$$\lim_{\upsilon\to 0}\frac{1-\gamma}{\upsilon^2} = -\frac{1}{2}$$

was used.

In the PF one gets

$$\dot{\mu}_{ik}\Big|^{PF} = \frac{d}{d\tau}\mu_{ik}\Big|^{PF} = \left.\begin{pmatrix} 0 & -\vec{A} \\ \vec{A} & (0)_{3\times 3}\end{pmatrix}\right|^{PF}, \quad \vec{A} = \vec{a}\times\vec{m}.$$

$$\ddot{\mu}_{ik}\Big|^{PF} = \left.\begin{pmatrix} 0 & -\vec{B} \\ \vec{B} & C_{3\times 3}\end{pmatrix}\right|^{PF}, \quad \vec{B} = (\vec{a}'\times\vec{m})\Big|^{PF}, \quad C_{3\times 3} = \left(c_{\alpha\beta}\right),$$

$$c_{\alpha\beta} = -\varepsilon_{\alpha\beta\delta}D_\delta, \quad \alpha,\beta = 1,2,3; \quad \vec{D} = \left(a^2\vec{m} - \vec{a}\left\langle\vec{m}\,\vec{a}\right\rangle\right)\Big|^{PF}.$$

APPENDIX *C*

Some obvious equality:

$$\vec{m}^2\Big|_{PF} = (-1/2)\mu^{nm}\mu_{mn};$$

$$\left\langle\vec{m}\,\vec{a}\right\rangle^2\Big|^{PF} = \mu^{nm}\omega_m\,\mu_{nk}\omega^k + (1/2)\mu^2\omega^2;$$

$$\left\langle\vec{m}\,\vec{a}'\right\rangle^2\Big|^{PF} = (\mu^{nm}\dot{\omega}_m)^2 + (1/2)\mu^2\dot{\omega}^2 - (1/2)\mu^2\omega^4;$$

$$m^\alpha\left\langle\vec{m}\,\vec{a}'\right\rangle\Big|^{PF} = \left[\mu^{\alpha m}\mu_{mn}\dot{\omega}^n - (1/2)\mu^2\dot{\omega}^\alpha\right]\Big|^{PF},$$

$$m^\alpha\left\langle\vec{m}\,\vec{a}\right\rangle\Big|^{PF} = \left[\mu^{\alpha m}\mu_{mn}\omega^n - (1/2)\mu^2\omega^\alpha\right]\Big|^{PF},$$

$$m^\alpha(\vec{a}'\times\vec{a})\Big|^{PF} = -u^l\dot{\mu}_{lk}\dot{\omega}^k u^\alpha, \qquad \alpha = 1,2,3;$$

$$\left\langle\vec{m}\,\vec{a}\right\rangle\left\langle\vec{m}\,\vec{a}'\right\rangle\Big|^{PF} = u_i\dot{\mu}^{im}\mu_{mn}\dot{\omega}^n + (1/2)\mu^2\left\langle\omega\,\dot{\omega}\right\rangle.$$

Vice versa, one can express 4-dimensional relations using 3-vectors as follows:

$$\mu^{nm}\omega_m\,\mu_{nk}\omega^k = (\mu^{nm}\omega_m)^2 = \left[\left\langle\vec{m}\,\vec{a}\right\rangle^2 - \vec{m}^2a^2\right]\Big|^{PF};$$

$$\mu^{nm}\dot{\omega}_m\,\mu_{nk}\dot{\omega}^k = \left(\mu^{nm}\dot{\omega}_m\right)^2 = \left[\left\langle\vec{m}\,\vec{a}'\right\rangle^2 - \vec{m}^2a'^2\right]\Big|^{PF};$$

$$\dot{\mu}^{nm}\dot{\omega}_m\,\dot{\mu}_{nk}\dot{\omega}^k = \left(\dot{\mu}^{nm}\dot{\omega}_m\right)^2 = -a^4\left(\vec{a}\times\vec{m}\right)^2 + \left\langle\vec{a}'\left(\vec{a}\times\vec{m}\right)\right\rangle^2\Big|^{PF};$$

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$$u_i \dot{\mu}^{im} \dot{\omega}_m = -\langle \vec{a}' (\vec{a} \times \vec{m}) \rangle \Big|^{PF};$$
$$\omega_i \mu^{im} \mu_{mn} \dot{\omega}^n = \vec{m}^2 \langle \vec{a}\, \vec{a}' \rangle - \langle \vec{m}\, \vec{a} \rangle \langle \vec{m}\, \vec{a}' \rangle \Big|^{PF};$$
$$\mu^{im} \mu_{mn} \omega^n \Big|^{PF} = \left(0,\ \vec{m} \langle \vec{m}\, \vec{a} \rangle - \vec{a}\vec{m}^2 \right) \Big|^{PF};$$
$$\mu^{im} \mu_{mn} \dot{\omega}^n \Big|^{PF} = \left(0,\ \vec{m} \langle \vec{m}\, \vec{a}' \rangle - \vec{a}'\vec{m}^2 \right) \Big|^{PF}.$$
$$\ddot{\mu}^{im} \mu_{mn} \dot{\omega}^n \Big|^{PF} = \left(\langle \vec{m}\, \vec{a}' \rangle^2 - \vec{m}^2 a'^2,\ \vec{m} \left[a^2 \langle \vec{m}\, \vec{a}' \rangle - \langle \vec{a}\, \vec{a}' \rangle \langle \vec{m}\, \vec{a} \rangle \right] - \vec{a}' \left[\vec{m}^2 a^2 - \langle \vec{m}\, \vec{a} \rangle^2 \right] \right) \Big|^{PF}.$$

APPENDIX *D*

Let us consider a charged particle with a magnetic moment moving uniformly along the circumference of a circle in a Lab frame. Suppose that the magnetic moment remains normal to the plane of the circle. If one directs the Ox axis along the instantaneous velocity $\vec{\upsilon}$ (at the certain time moment) then

$$a_x = a_z = 0, \quad a_y = -\upsilon^2/R, \quad a_x' = da_x/dt = -\upsilon^3/R^2, \quad a_y' = a_z' = 0,$$

and, according to App. A, we find:

$$\omega^i = (0, 0, -\upsilon^2\gamma^2/R, 0), \ \ \dot{\omega}^i = (0, -\upsilon^3\gamma^3/R^2, 0, 0).$$

According to (3.5), we have

$$\vec{m} = (0, 0, |\vec{m}|\gamma).$$

In the PF (according to Lorentz transformations) we get:

$$\omega^{i\,PF} = (0, 0, -\upsilon^2\gamma^2/R, 0), \qquad \dot{\omega}^{i\,PF} = (\upsilon^4\gamma^4/R^2, -\upsilon^3\gamma^4/R^2, 0, 0).$$

So, according to App. A, we obtain:

$$\vec{a}^{\,PF} = (0, -\upsilon^2\gamma^2/R, 0), \quad \vec{a}'^{\,PF} = (d\vec{a}/dt)\Big|^{PF} = (-\upsilon^3\gamma^4/R^2, 0, 0);$$
$$\vec{m}^{\,PF} = (0, 0, |\vec{m}|).$$

According to (5.15) and (5.16) one has

$$F^{i\,PF} = (F^0, \vec{F})\Big|^{PF}; \quad F^{0\,PF} = 0, \ \ \vec{F}^{\,PF} = \left((2e^2/3)\vec{a}' + (2e/3)a^2(\vec{a}\times\vec{m}) + (52/105)\vec{a}'a^2\vec{m}^2\right)\Big|^{PF} \quad \Rightarrow$$
$$F^{i\,PF} = \left(0,\ -\left(2e^2/3 + (2e/3)|\vec{m}|\upsilon^3\gamma^2R^{-1} + (52/105)m^2\upsilon^4\gamma^4R^{-2}\right),\ 0,\ 0\right)\cdot\upsilon^3\gamma^4/R^2.$$

The expression for the last force in the Lab frame one gets using Lorentz transformations:

$$F^i = -\gamma^5 \frac{\upsilon^3}{c^3}\left(\frac{\upsilon}{c}, \frac{\vec{\upsilon}}{\upsilon}\right)\cdot\frac{1}{R^2}\left(2e^2/3 + (2e/3)(|\vec{m}|/R)\gamma^2(\upsilon^3/c^3) + (52/105)(\vec{m}^2/R^2)\gamma^4(\upsilon^4/c^4)\right).$$

Here, as usual, $c = 3\cdot 10^8\ m/s$.

REFERENCES

[1] F. Rohrlich, Classical Charged Particle, 3rd ed., World Scientific Publishing Co. Pte. Ltd., 2007.

[2] C. Teitelboim, “Splitting of the Maxwell Tensor: Radiation Reaction without Advanced Fields”, Phys. Rev. D, 1970, v. 1, **6**, pp.1572-1582.

[3] Б. Косяков. “Излучение в электродинамике и теории Янга-Миллса”, УФН, 1992, т. 162, №2, стр. 161-176.

[4] *a*) N. Chachava, I. Lomidze, D. Karkashadze, J. Javaxishvili, “About One Contradiction in Classical Electrodynamics”, Proceedings of I. Javaxishvili Tbilisi State University, 2004, 357, Physics, **39**;

ISSN 1512-1461

b) N. Chachava, I. Lomidze, D. Karkashadze, J. Javaxishvili, “About One Contradiction in Classical Electrodynamics”, JINR Communications E2-2005-180, Dubna, 2005.
[5] L. Landau, E. Lifshitz, Course of Theoretical Physics, V. 2, Fourth Revised English Edition.
[6] R. Feynman, R. Leighton, M. Sands. The Feynman Lectures on Physics. Caltech, 2013.
[7] J. Frenkel, “Die Elektrodynamik des rotierenden Elektrons”, Zeitschrift fur Physik, 1926, V. 37 (Rus. Transl.: Я. Френкель, “Электродинамика вращающегося электрона”, Собр. Избр. трудов, т. 2, стр. 460-476, М., Изд-во Акад. Наук СССР, 1958).
[8] P. Dirac, Classical Theory of Radiation Electrons, Proc. Roy. Soc. 1938 (London), A167, 148.
[9] W. Panofsky, M. Phillips, Classical Electrissity and Magnetism, 2nd ed., Addison-Wesley Publ. Co., 1929.
[10] М. Лаврентьев, Б. Шабат, Методы теории функций комплексной переменной, М., «Наука», 1973.
[11] *a*) Burn, B. J. "A synchrotron model for the continuum spectrum of the Crab Nebula". Monthly Notices of the Royal Astronomical Society, 1973, 165 (4), 421-429;
b) D. Volpi at al., Non-thermal emission from relativistic MHD simulations of pulsar wind nebulae: from synchrotron to inverse Compton, Astronomy&Astrophysics, 2008, v. 485, **2**, p. 337-349.

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