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INSTABILITY OF THERMOMAGNETIC WAVES IN ANISOTROPIC MEDIA

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Abstract

It is shown that when choosing the crystal symmetry, if the wave vector of the wave and the temperature gradient are oriented arbitrarily, waves of a thermomagnetic nature of the same frequency and growth rate are possibly excited. During hydrodynamic motions of a non-equilibrium plasma, which are in a constant temperature gradient (i.e. $\nabla T = const$), an oscillation of physical quantities is excited, and the plasma has oscillatory properties. At the same time, plasma is very different from ordinary plasma. In such a plasma, a transverse thermomagnetic wave is excited $\vec{k} \parallel \vec{\nabla} T$, in which only the magnetic field oscillates. When there is an external magnetic field, the wave vector of thermomagnetic waves is perpendicular to the magnetic field. In such a plasma, a weak magnetic field arises during hydrodynamic motion, i.e. $\Omega\tau << 1$. (Ω-Larmar frequency of electrons, τ-time of collision of electrons). In these media, the Nernst-Ettingishausen coefficient and the differential thermoelectric power depend very strongly on the electric field voltage. Theoretical studies of excited thermomagnetic waves in the above media found the oscillation frequency of the corresponding physical quantities and the growth rate of these oscillations in certain directions of the crystals.

***Keywords**: frequency, increment, excitation, symmetry, tensor, orientation.*

Introduction

It was shown in [1] that hydrodynamic motions in a nonequilibrium plasma, in which there is a constant temperature gradient ($\nabla T = const$), has oscillatory properties. This property of plasma is very different from ordinary plasma. Without an external magnetic field and hydrodynamic motions, transverse "thermomagnetic" waves are possible in it, in which only the magnetic field oscillates. In the presence of an external magnetic field, the wave vector of thermomagnetic waves is perpendicular to the magnetic field and lies in the plane ($\vec{H}, \vec{\nabla} T$). If a weak magnetic field appears in such a plasma, i.e. $\Omega\tau << 1$ (Ω-Larmar frequency of electrons, τ-time of collision of electrons) inside the plasma arises in addition to the external electric field, the electric is proportional to the temperature gradient, the electric field is proportional to the magnetic field. Due to this complex electric field, thermomagnetic waves of a transverse $\vec{k} \perp \vec{\nabla} T$ ($\vec{k}$-wave vector) and longitudinal character $\vec{k} \parallel \vec{\nabla} T$ are excited. A theoretical study of these thermomagnetic waves in isotropic conducting media of the electric type of charge carriers was carried out in [2-5]. However, in anisotropic conducting media, there is no theoretical study of thermomagnetic waves. In this theoretical work, we will investigate thermomagnetic waves in anisotropic conducting media with selected samples.

Basic equations of the problem

In the presence of an external magnetic field and a temperature gradient in an isotropic solid, the total electric field has the form

$$\vec{E} = \xi \vec{j} + \xi' \left[\vec{j}\vec{H} \right] + \xi'' \left(\vec{j}\vec{H} \right) \vec{H} + \Lambda \frac{\partial T}{\partial x} + \Lambda' \left[\vec{\nabla} T \vec{H} \right] + \Lambda'' \left(\vec{\nabla} T \vec{H} \right) \vec{H} \quad (1)$$

$\vec{j}$ - current flux density

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In anisotropic conducting media, all coefficients in equation (1) are tensors.

Then the equations for anisotropic conducting media will have the form:

$$E_i = \xi_{ik}\vec{j}_k + \xi'_{ik}\left[\vec{j}\vec{H}\right]_k + \xi''_{ik}\left(\vec{j}\vec{H}\right)H_k + \Lambda_{ik}\frac{\partial T}{\partial x_k} + \Lambda'_{ik}\left[\vec{\nabla}T\vec{H}\right]_k + \Lambda''_{ik}\left(\vec{\nabla}T\vec{H}\right)H_k \quad (2)$$

Here ξ_{ik} is the tensor of the reciprocal of the ohmic resistance, Λ_{ik} is the differential thermoelectric power, and Λ'_{ik} is the Nernst-Ettinishausen coefficient. We will consider a solid external magnetic field $\vec{H}_0 = 0$. Then, in the equations, the terms containing are equal to zero. Taking into account Maxwell's equation, we obtain the following system of equations

$$\begin{cases} E'_i = \xi_{ik} j'_k + \Lambda'_{ik}\left[\nabla TH\right]_k \\ rot\vec{E}' = -\frac{1}{c}\frac{\partial \vec{H}'}{\partial t} \\ rot\vec{H}' = \frac{4\pi}{c}\vec{j}' + \frac{1}{c}\frac{\partial \vec{E}'}{\partial t} \end{cases} \quad (3)$$

Assuming that all variable quantities are monochromatic in nature, i.e.

$$\left(E', H', j\right) \sim e^{i\left(\vec{k}\vec{r} - \omega t\right)} \quad (4)$$

From (3) it turns out:

$$E'_i = \xi_{ik} j'_{ik} + \Lambda'_{ik}\left[\vec{\nabla}T\vec{H}\right]_k \quad (5)$$

$$j'_k = \frac{ic^2}{4\pi\omega}\left[\vec{k}\left[\vec{k}\vec{E}'\right]\right]_k + \frac{i\omega}{4\pi}E'_k$$

(ω-oscillation frequency).

Theory

We (5) it turns out:

$$E'_i = \frac{ic^2}{4\pi\omega}\xi_{ik}\left(\vec{k}\vec{E}'\right)k_k + \frac{i\omega^2 - ic^2k^2}{4\pi\omega}\xi_{ik}E'_k + \frac{c\Lambda'_{ik}}{\omega}\left(\vec{\nabla}TE'\right)k_k - \frac{c\Lambda'_{ik}}{\omega}\left(k\vec{\nabla}T\right)\vec{E}' \quad (6)$$

To obtain the dispersion equation from (6), you first need to choose a coordinate system. We will choose the following coordinate system

$$k_1 \neq 0,\ k_2 = k_3 = 0,\ \frac{\partial T}{\partial x_2} \neq 0,\ \frac{\partial T}{\partial x_3} = 0 \quad (7)$$

Taking into account (7), from (6) it turns out::

$$E'_i = \left(A\xi_{ik}k_e k_k + B\xi_{ik} + \frac{c\Lambda'_{ik}}{\omega}k_e\frac{\partial T}{\partial x_k}\right)E'_k \quad (8)$$

When obtaining (8) from (7), we assumed that $\vec{k} \perp \vec{\nabla}T$ i.e. the resulting thermomagnetic waves are transverse.

At $E'_i = \delta_{ik}E'_k$, $\delta_{ik} = \begin{cases} 1, i = k \\ 0, i \neq k \end{cases}$ (9)

From (8) we obtain the following dispersion equations in tensor form

$$N_{ik} = A\xi_{ie}k_e k_k + B\xi_{ik} + \frac{c\Lambda'_{ie}}{\omega}k_e\frac{\partial T}{\partial x_k} \quad (10)$$

$$A = \frac{ic^2}{4\pi\omega},\quad B = \frac{i\omega^2 - ic^2k^2}{4\pi\omega}$$

Expanding by components (10) it turns out:

$$\begin{aligned} \left|\left(N_{ik} - \delta_{ik}\right)\right| &= \left(N_{11} - 1\right)\left(N_{22} - 1\right)\left(N_{33} - 1\right) + \\ &+ N_{31}N_{12}N_{23} + N_{21}N_{32}N_{13} - N_{31}N_{13}\left(N_{22} - 1\right) - \\ &- N_{32}N_{23}\left(N_{11} - 1\right) - N_{21}N_{12}\left(N_{33} - 1\right) = 0 \end{aligned} \quad (11)$$

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Here:

$$N_{11}=\frac{i\omega}{4\pi}\xi_{11},\quad N_{12}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{12}+4\pi c\Lambda'_{11}k\nabla_2T}{4\pi\omega},\ N_{13}=\frac{i\omega^2-ic^2k^2}{4\pi\omega}\xi_{13},\ N_{21}=\frac{i\omega}{4\pi}\xi_{21},$$

$$N_{22}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{22}+4\pi c\Lambda'_{21}k\nabla_2T}{4\pi\omega},\ N_{23}=\frac{i\omega^2-ic^2k^2}{4\pi\omega}\xi_{23},\ (12)$$

$$N_{31}=\frac{i\omega}{4\pi}\xi_{31},\ N_{32}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{32}+4\pi c\Lambda'_{31}k\nabla_2T}{4\pi\omega},\ N_{33}=\frac{i\omega^2-ic^2k^2}{4\pi\omega}\xi_{33}$$

Putting (12) into (11), we obtain the following equation for the oscillation frequency inside an anisotropic body

$$\Phi_5\omega^5+\Phi_4\omega^4+\Phi_3\omega^3+\Phi_2\omega^2+\Phi_1\omega+\Phi=0\ (13)$$

Solution (13) in a general form is not possible and therefore we will not write out the expression $\Phi,\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5$.

To solve the dispersion equation (11), we will choose crystals satisfying the following conditions.

1) $N_{11}=N_{21}=N_{31}$, 2) $N_{21}=N_{32}=N_{22}$ 3) $N_{13}=N_{23}=N_{33}$ (14)

A crystal satisfying conditions (14) is diagonal.

Taking into account (14), from (11) we easily obtain

$$N_{11}+N_{22}+N_{33}=1\ (15)$$

If

$$\xi_{11}=\xi_{22}=\xi_{33}=\xi\quad(16)$$

from (15) we get

$$\frac{3}{4\pi}\xi\omega^2-\frac{c^2k^2}{2\pi}\xi+i\omega'_{21}=0\ (17)$$

$$\omega'_{21}=-c\Lambda'_{21}k_1\nabla_2T$$

From (17) we get:

$$\omega=\omega_0+i\gamma=\frac{\sqrt{2}}{3}\left(\frac{6\pi|\omega'_{21}|}{\xi}+c^2k^2\right)^{1/2}+i\frac{\sqrt{2}}{3}\left(\frac{6\pi|\omega'_{21}|}{\xi}-c^2k^2\right)^{1/2}\ (18)$$

It can be seen from (18) that at $c^2k^2>\frac{6\pi\omega'_{21}}{\xi}$, the exciting wave is of a purely electromagnetic nature.

When $c^2k^2\xi<6\pi|\omega'_{21}|$ the excited wave is growing thermomagnetic with a frequency

$$\omega_0=\left(\frac{4\pi\omega'_{21}}{\xi}\right)^{1/2}\left(1+\frac{1}{12\pi}\frac{c^2k^2\xi}{|\omega'_{21}|}\right)\text{ and increment }\gamma=\left(\frac{4\pi\omega'_{21}}{\xi}\right)^{1/2}\left(1-\frac{c^2k^2\xi}{12\pi|\omega'_{21}|}\right)$$

For $\vec{k}\parallel\vec{\nabla}T$ from (11), consider the case

1) $N_{11}=N_{21}=N_{31}$, 2) $N_{13}=N_{32}=N_{33}=N_{23}=N_{13}$ (19)

$$N_{11}=\frac{i\omega}{4\pi}\xi_{11},\quad N_{12}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{12}-4\pi c\Lambda'_{12}\vec{k}_1\vec{\nabla}_2T}{4\pi\omega},\ N_{13}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{13}-4\pi c\Lambda'_{12}\vec{k}\vec{\nabla}T}{4\pi\omega},$$

$$N_{21}=\frac{i\omega}{4\pi}\xi_{21},\quad N_{22}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{22}-4\pi c\Lambda'_{22}\vec{k}\vec{\nabla}T}{4\pi\omega},\ N_{23}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{23}-4\pi c\Lambda'_{23}\vec{k}\vec{\nabla}T}{4\pi\omega},$$

(20)

$$N_{31}=\frac{i\omega}{4\pi}\xi_{31},\quad N_{32}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{32}-4\pi c\Lambda'_{32}\vec{k}\vec{\nabla}T}{4\pi\omega},\ N_{33}=\frac{\left(i\omega^2-ic^2k^2\right)\xi_{33}-4\pi c\Lambda'_{33}\vec{k}\vec{\nabla}T}{4\pi\omega}$$

Taking into account (19-20) from (11) we get:

$$N_{11}-1+2N_{11}N_{22}=0\ (21)$$

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$$\xi_{11} = \xi_{22} = \xi$$

$$\frac{i\omega\xi}{4\pi} - \frac{\xi^2\omega^2}{8\pi^2} + \frac{\xi^2 c^2 k^2}{8\pi^2} - \frac{i\omega'_{21}\xi}{2\pi} - 1 \quad (22)$$

From solution (22) we obtain

$$\omega_1 = \frac{i\pi}{\xi} - \left(\frac{2\omega'_{21}\xi}{\pi}\right)^{1/2}\frac{\pi}{\xi} + \frac{i\pi}{\xi}\left(\frac{2\omega'_{21}\xi}{\pi}\right)^{1/2} \quad (23)$$

$$\omega_2 = \frac{i\pi}{\xi} + \left(\frac{2\omega'_{21}\xi}{\pi}\right)^{1/2}\frac{\pi}{\xi} - \frac{i\pi}{\xi}\left(\frac{2\omega'_{21}\xi}{\pi}\right)^{1/2}$$

From (23) it can be seen that a wave with a frequency $\omega_0 = -\left(\frac{2|\omega'_{21}|}{\pi\xi}\right)^{1/2}$ is increasing, a wave with a frequency $\omega_0 = -\left(\frac{2|\omega'_{21}|}{\pi\xi}\right)$ can grow if $2|\omega'_{21}|\xi < \frac{\pi}{2}$

For an arbitrary orientation of the wave vector relative to the temperature gradient, from tensor (10), we obtain

$$N_{11} = \frac{i\omega^2\xi_{11} + \omega_{11}}{4\pi\omega}, \quad N_{12} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{12} - \omega_{11} + \omega_{12}}{4\pi\omega}, \quad N_{13} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{13} + \omega_{13}}{4\pi\omega},$$

$$N_{21} = \frac{i\omega^2\xi_{21} + \omega_{21}}{4\pi\omega}, \quad N_{22} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{22} + \omega_{22}}{4\pi\omega}, \quad N_{23} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{23} + \omega_{23}}{4\pi\omega}, \quad (24)$$

$$N_{31} = \frac{i\omega^2\xi_{31} + \omega_{31}}{4\pi\omega}, \quad N_{32} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{32} + \omega_{32}}{4\pi\omega}, \quad N_{33} = \frac{\left(i\omega^2 - ic^2k^2\right)\xi_{33} + \omega_{33}}{4\pi\omega},$$

$$\omega_{ik} = -4\pi c\Lambda'_{ik}\left(\vec{k}\vec{\nabla}T\right)$$

Choosing a crystal

From (11), taking into account (24), we obtain the following dispersion equation for determining the frequency and growth rate of the excited waves inside an anisotropic crystal

$$\frac{i\omega^2\xi_{11} - \omega_{11}}{4\pi\omega}\left(\frac{i\omega^2\xi_{12} - ic^2k^2\xi_{12} - \omega_{11} - \omega_{12}}{4\pi\omega} + 1\right) = \frac{i\omega^2\xi_{22} - ic^2k^2\xi_{22} + \omega_{22}}{4\pi\omega}\left(\frac{i\omega^2\xi_{11} - \omega_{11}}{4\pi\omega} - 1\right) \quad (25)$$

At $\omega_{12} = \omega_{11} + \omega_{22}$ and $\xi_{11} = \xi_{22} = \xi_{12}$ from (25) we get

$$\omega^2 = \frac{1}{2}\left(c^2k^2 + i\frac{\omega_{12}}{\xi}\right) \quad (26)$$

Solution (26) gives

$$\omega = \frac{ck}{2}\left(\frac{\omega_{12}}{c^2k^2\xi}\right)^{1/2}(1+i) \quad (27)$$

or $\omega = \omega_0 + i\gamma$

$$\omega_0 = \left(\frac{\omega_{12}}{4\xi}\right)^{1/2}, \quad \gamma = \left(\frac{\omega_{12}}{4\xi}\right)^{1/2} \quad (28)$$

For any orientation of the wave vector with respect to the temperature gradient, the frequency and growth rate of the excited thermomagnetic are the same.

Discussion of the results

In anisotropic conducting media of the electric type of charge carriers in an external electric field in the presence of a constant temperature gradient, longitudinal $\vec{k} \parallel \vec{\nabla}T$ and transverse $\vec{k} \perp \vec{\nabla}T$ waves of a thermomagnetic nature are excited. The frequency and growth rate of this wave depend

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on the conductivity of the medium. The conductivity of a medium is easily expressed in terms of the diagonal values of the conductivity. This creates a favorable condition for experimental verification of the exciting waves. If the wave vector of the excited waves has an arbitrary direction relative to a constant temperature gradient, then the frequency and growth rate have the same values. When calculating, we choose crystals of different symmetry. Of course, the conditions for the excitation and growth of the wave will be different if we choose different symmetries from the tensor (11).

Conclusion

It is proved that in anisotropic conducting media of electric type of charge carriers, different waves of a thermomagnetic nature are excited. With the longitudinal $\vec{k} \parallel \vec{\nabla} T$ and transverse $\vec{k} \perp \vec{\nabla} T$ orientation of the wave vector relative to the temperature gradient, waves of a thermomagnetic nature with different frequencies and increments are excited.

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