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# THE DEVELOPMENT OF MATHEMATICAL MODEL FOR THE DEFINATION OF DISASTROUS WAVES PRODUCING IN THE WATER BODIES

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**Abstract**

*The negative consequences caused by the natural processes on our entire planet, including in Georgia, have reached colossal proportions. The damage to the economy amounts to tens of billions of dollars every year, human toll is also high. The territory of Georgia belongs to the most difficult region of the mountainous ones of our planet due to the diverse range of natural processes, scale of development, repeatability over time, negative consequences for the population, agricultural fields and agricultural-engineering facilities. The entire territory of our country, more or less damaged by natural processes is in the risk zone of their impact. Landslides, rock falls, mudslides, washing out of the shores of the sea and erosion reservoirs banks, water falls, rock falls, karst-suphosic subsidence and the socio-economic consequences caused by them cover all fields of human activity.*

Key words: Reservoir, comparative, slope, stress, flash- flow, dam, disaster.

## Introduction

The nature of the course of geodynamic processes in the mountainous regions, the increase of the slope stress field and the danger of the large-scale development of gravity events are unimaginably increased by the fact that the entire territory of the country is in the risk zone of 7-9 earthquakes. The situation is further complicated by the fact that, for the most part, in the same territory, the simultaneous occurrence and activation of various types of events take place (as it happened in august 2023 during the tragedy in Shovi recreation complex in Racha, in the result of which 32 persons were died and the infrastructures were destructed, also in a month later in Guria in case of mud stream 3 childes were died), due to which it is very difficult to predict individual events and to plan remedial measures in the area.

Method of comparative calculations presented in the article is made of the approximate Mathematical model developed by us. This methodology will see the spread of waves generated in different types of water bodies, for example reservoirs like Gumati and Vartsikhe ones, (riv.Rioni basin) coastal waters and estuaries (earthquakes, landslides, sale streams and other events) and their influence on different kind of dams, as well as other hydraulic facilities.

The results of the performed calculations and Mueller's data, fully demonstrate the reliability of our methodology. Here presented model has to be successfully used for the disaster event prediction in different mountainous river basins of the world .

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The results of the theoretical calculation method are compared to the data on the Viont (Italy) disaster obtained by L. Muller. This disaster can be considered the only real object that coincides with the task discussed in many parameters.

The purpose of the comparative calculations below is to compare the data obtained by theoretical methods with natural data.

Due to the mathematical difficulties arising in the description of disastrous events, the first -line task of the invading mountain rocks is the physical modeling of this process, which enables us to believe (or reject) the legality and consideration of the mathematical hypotheses and opinions selected, on the basis of which the mathematical model of the event is established [1]. However, it is important to note that this kind of physical modeling is a very difficult task due to the scale and complexity of the processes to be considered [2].

The implementation of the so-called "calculating modeling" is essential to this, in which the theoretical attitudes developed for the object studied are tested on such events that have already occurred on object-analogs, which are well known and described quite fully.

The object for a such a "calculating model" is used in the fact of invasion of mountain rocks in the Wayon Reservoir [3]. This landslide happened in 1963 at the foot of Mount Monte-Toki (the Piev River). The rocks invaded from the left side of the valley in the reservoir. As a result, the produced waves height on the north slope reached 260-m from reservoir mean level at the village of Caso. The waves of 100 m. high were moved over the dam. They led to a flood that completely destroyed five villages in the in the valley of river Piav. No eyewitnesses of the disaster) were survived (disaster occurred at about 11 pm). The data of the wave sizes resulting from the invasion of the ground masses in the reservoir were collected after the disaster by the traces left by the water flow.

It is important to note that catastrophic events, due to the difference between the causes of induced wave processes, as well as the geometric dimensions and the difference between the waterbodies and the difference in configuration, can be completely different. Specifically, the wave processes can be induced by horizontal and vertical displacement of the reservoir slopes and seabed, landslide and ground masses falling into the water, etc [4].

The wave movement developed by disastrous events and its accompanying to these ones are very diverse and different. Our goal is through the "calculating modeling", confirm the suitability of the selected mathematical model. Because of this, we will discuss the events that can only be verified and compared to Mueller data when invading different arrays in the water bodies. Obviously we will not touch the processes that accompany such catastrophic events, and which cannot be verified due to the lack of data.

Below we have presented the natural data described and preserved on the disaster of Vaiont disaster. At the end of the article we will bring the results of the calculations we have performed by these events.

For "calculating modeling" (according to Muller) we have the so-called four "fulcrum points": 1) When landslides, mudslides and other types of landslides invade the water body, their movement parameters are determined taking into account such features, during which we consider the movement as a principle, when the massif of landslides is partially underwater even before the movement begins. So, it is essential to consider the movement of this massif while moving simultaneously on the underwater and on the surface slopes. In such an approach, along with the Coulon frictional forces, the existence of frontal hydrodynamic forces, coupled water masses and wave resistance forces are considered [5].

In the case of such an approach, the one-dimensional equation of the the dynamics of different types of rocks in the water bodies has the following look:

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$$V_{S1}^2 = V_{S0}^2 + \left(A_1 - 2A_0V_{S0}^2\right)\xi - \left(2A_0A_1 + A_1A_3 + A_2 + A_3V_{S0}^2\right)\frac{\xi^2}{2} + 2A_0A_2\frac{\xi^3}{6} + A_2A_3\frac{\xi^4}{8}, \quad (1)$$

Where $V_s$ - is the speed of the mass center of the array, the volume of which is equal to $W_{S0}$; $\xi = \frac{l_{sw}}{L_s}$, $l_{sw}$ - is the length of mountain rocks under water; $L_s$ - full length of moving mountain rocks;

$$A_0 = \beta\frac{\rho_w}{\rho_s};\ A_1 = 2gI_{ss}L_s;\ A_2 = 2gL_s\left(I_{ss} - \sigma_s I_{sw}\right);\ A_3 = \frac{3}{2}\bar{k}_w\frac{\rho_w}{\rho_s};\ \bar{k}_w = k_w + \lambda_w\frac{L_s}{L_{sm}};$$

$\beta$ -is attached mass coefficient; $\rho_s$ and $\rho_w$ - densities of mountain rocks and water;

$$I_{ss} = \left(1 + k_c f_s\right)\sin\psi_s - f_s\cos\psi_s + k_c\cos\psi_s;$$
$$I_{sw} = \left(1 + k_c f_{sw}\right)\sin\psi_{sw} - f_{sw}\cos\psi_{sw} + k_c\cos\psi_{sw};$$

$k_c$ - Coefficient of seismicity. in our case =0;

$k_w$ - forehead hydrodynamic resistance coefficient;

$f_s$ and $f_{sw}$ - Coulon friction coefficients respectively for those slopes that are located under water and above water;

$\psi_s$ - the angle of inclination of the sliding surface of the mountain massif (located above the water table), $\psi_{sw}$ - the same one located underwater $\sigma_s = l - \frac{\rho_w}{\rho_s}$; $\lambda_w$, - the linear hydraulic resistance coefficient of the mountain massif's friction with water, $h_{sm}$ - the maximum height of the massif.

The main characteristics of the forced waves generated in the aquifer are determined by the parameters of the intrusive rocks given in relation (1).

In this case, we have 1 "support" point: the maximum mark of the massif of mountain rocks intruded into the reservoir was m=120 m; 107 m was obtained by calculation; The difference is 12%.

2) When determining the characteristics of forced waves generated by catastrophic events, different types of wave theories are used: a) when the waves are long, that is, the inequality $\frac{\lambda}{H} \gg 1$ is fulfilled ($\lambda$ - wavelength is, $H$ -aquifer depth), then it is correct to use the theory of linear long waves; b) when the amplitudes of long waves and wave velocities are not small, nonlinear equations of long waves suitable for wave motion with finite height will be used; c) in cases where the condition is fulfilled in the reservoir $\frac{h}{\lambda} = \varepsilon$ : it is small (thin water waves), a system of equations will be used, where frictional forces are taken into account; d) Voynich-Sianozhentsky local non-stationarity theory, which was developed for cases of intrusion of landslide-sand flows into aquifers. This theory is based on the system of equations of unsteady one-dimensional motion of a real fluid, i.e. the system of Saint-Venant-Bussinesq equations; e) Finally, computational dependences of forced wave characteristics when only energy and mass conservation laws are used. This approach is used in our case. Note that the relationships are derived theoretically and do not contain any empirical coefficients. This approach is fair for both short and long waves.

In this case, the energy balance equation of wave motion is used:

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$$\frac{\partial E}{\partial t}+\frac{\partial (EU)}{\partial x}=N-D, \qquad (2)$$

Where $E$ is the wave energy;

$U$ -group speed of waves $\qquad U=\frac{c}{2}\left(1+\frac{2kH}{sh2kH}\right), \qquad (3)$

Here $c$ is the phase speed of the waves; $H$ - water depth; $k$ -wavenumber $k=\frac{2\pi}{\lambda}$, $\lambda$ - wavelength; $N$ - intensity of external energy flow along the $x$ axis; $D$ - Dissipation of wave energy.

Equation (2) firstly was introduced into the theory of surface waves by Makaveev, then by Sverdrup and Munk. After integration of (2), for the characteristics of forced waves, we get the following relationships: for wave height, length and phase speed:

$$\left.\begin{aligned} h&=\frac{8}{\pi}\frac{\rho_s}{\rho_w}\frac{B_s}{B_w}\left(\frac{V_s^2}{2g}+H_s\right),\\ \lambda&=\frac{\pi V_s H_s t}{h},\\ C_0&=\sqrt{\frac{g\lambda}{2\pi}th\frac{2\pi H_w}{\lambda}}, \end{aligned}\right\} \qquad (4)$$

and for the crest length of the waves:

$$B_w=2t\left(C_0+V_s\right)\left\{1+\frac{2}{3}\left[\frac{c_0 t}{\left(c_0+V_s\right)}\right]^2-\frac{2}{5}\left[\frac{c_0 t}{\left(c_0+V_s\right)}\right]^4\right\} \qquad (5)$$

where, $V_s$ - is the speed of the center of mass of the landslide body;

$B_s$ - the length of the landslide front; опорная точка point

$H_s$ - The thickness of the landslide body underwater.

In case 2), when determining the parameters of the waves, we have two " fulcrum point": the height $h$ of the overflowing wave in the center of dam and its right edge in m [6].

3) The determination of the height of the wake flow of forced waves on the slopes of the reservoir is based on the equation of the speed of propagation of the crushed thallus front, in which the Stokes transfer speed is neglected compared to the phase speed:

$$\frac{dx}{dt}=\sqrt{\frac{g}{k}thkH}\left(l-\frac{x}{e}\right)^p\cos\alpha, \qquad (6)$$

Here $k, H$ - are the characteristics of the breaking wave, $l$ -is the height of the surge flow rise on the slope; $p$ -represents the intensity of the influence of frictional forces on the speed of the breaking wave flow. The energy losses of the crushed wave are taken into account by the Shezy- Manning hydraulic relations [7]:

$$h_{\text{დან}}=\frac{V^2 l}{c_{\text{შ}}^2 H} \qquad (7)$$

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Finally, for the height of the gust flow, we get the calculation relation:

$$h = 0.25\left[\left(1+\frac{c}{U}\theta\right)\left(\varepsilon-\varepsilon_{\text{კრ}}\right)\right]\sqrt{2\pi\lambda_{\text{კრ}}H_{\text{კრ}}thkH_{\text{კრ}}}\,\frac{\sqrt[3]{H_{\text{კრ}}}\sin\alpha}{\sqrt[3]{H_{\text{კრ}}}+1.5gn^2} \qquad (8)$$

Here $\theta$ - is the symbol of the Heaviside [7] function, $\varepsilon = \frac{h}{\lambda}$ - is the wave steepness, $n$ - is the roughness coefficient.

## Conclusion

Below we present the results of the performed calculations and Mueller's data, which demonstrate the reliability of our methodology ( see table 1).

Table 1.

| | The overflow wave height on the dam center $h$ in m | The overflow wave height $h$ in m on the dam (right edge) | The height of the surge on the slope of the reservoir (near the village of Kasos) $h$ in m | The maximum mark of the mountain range intruding into the reservoir $h$ in m |
|---|---|---|---|---|
| natural data | 90 | 100 | 200 | 120 |
| Calculation results | 82.5 | 133 | 213 | 107 |
| The difference | 9% | 33 % | 6.5 % | 12 % |

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