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Z^0-BOSONS SCATTERINGS WITH EXCHANGE OF HIGGS SCALARS IN LADDER APPROXIMATION

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Abstract
In the standard $SU(2)\times U(1)$ theory of electroweak interaction with the nonperturbative regime, Z^0-bosons scattering with the exchange of Higgs bosons are studied. Using ladder diagrams approximations, Bethe-Salpeter type integral equations with minimally perturbative kernels for scattering amplitudes are formulated. In the Regge regions of energy changes, asymptotic solutions of the corresponding Bethe-Salpeter equations for the imaginary part of scattering amplitudes in the form of power functions are found.

Keywords: *spontaneous symmetry breaking, Higgs particles, nonperturbative approximation, Bethe-Salpeter equation*

Introduction

The creation of the mechanism for electroweak symmetry breaking (SB) which generates the masses of known elementary particles has been one of the fundamental efforts in particle physics for many years. The discovery in 2012 by the ATLAS [1] and CMS [2] collaborations of a new resonance with a mass of about *125 GeV* and subsequent studies of its properties with center-of-mass energies of *7 TeV* and *8 TeV*, convincingly presented the primary portrait of the mechanism of spontaneous symmetry breaking (SSB) of the electroweak interaction. Data collected during the launch of the Large Hadron Collider (LHC) with a higher center of mass energy of *13 TeV* and a more prominent data set firmly established the compatibility of the measured resonance with the Standard Model (SM) Higgs boson [3]. The launch of the third session of the LHC is expected to provide new data on proton-proton collisions.

As is known electroweak interactions in the Standard Model (SM) are described by a gauge theory that is invariant with respect to the symmetry group $SU(2)\times SU(1)$. The SB electroweak interaction mechanism [4] provides a general framework that allows the structure of these gauge interactions to be preserved intact at high energies, resulting in the generation of the observed masses of the $W^{\pm}$ and Z^0 gauge bosons. The mechanism postulates a self-acting complex doublet scalar field, for which the CP-even neutral component acquires a vacuum mathematical expectation (VME) $\upsilon = 246$ $ГЭВ$, which specifies the scale of SB. Three massless Goldstone bosons are generated and absorbed, giving mass to the gauge $W^{\pm}$ and Z^0 bosons. The remaining component of the complex doublet becomes the Higgs boson - a new and so far unique fundamental scalar particle. The masses of all fermions are also a consequence of the SNA of the electroweak

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interaction, since the Higgs doublet is assumed to couple to the fermions through Yukawa interactions. Remarkably, these Yukawa interactions with heavy fermions have already been established.

The connection between the Higgs boson and fundamental particles is determined by their masses. This new type of interaction is very weak for light particles such as u- and d-quarks and electrons, but strong for heavy particles such as W and Z bosons b- and t- quarks. More precisely, the SM Higgs couplings with fundamental fermions are linearly proportional to the fermion masses, while the coupling constant of the Higgs particle with bosons is proportional to the square of the boson masses. The interactions of the SM Higgs boson with gauge bosons and fermions, as well as with themselves, are summarized in the following Lagrangian:

$$L = -g_{Hf\bar{f}}\bar{f}fH + \frac{g_{HHH}}{6}H^3 + \frac{g_{HHHH}}{24}H^4 + \delta_G G_\mu G^\mu \left(g_{HGG}H + \frac{g_{HHGG}}{2}H^2 \right), \quad (1)$$

where

$$g_{Hf\bar{f}} \equiv y_f = \frac{m_f}{\upsilon}, \quad g_{HGG} = \frac{2m_G^2}{\upsilon}, \quad g_{HHGG} = \frac{2m_G^2}{\upsilon^2}, \quad g_{HHH} = \frac{3m_H^2}{\upsilon}, \quad g_{HHHH} = \frac{3m_H^2}{\upsilon^2}, \quad (2)$$

and

$$G = W^{\pm}, \text{ or } Z^0, \ \delta_W = 1, \ \delta_{Z^0} = 1/2 . \quad (3)$$

As a result, the dominant mechanisms of Higgs boson H production and decay include interactions with $W^{\pm}$- , Z^0- bosons and/or third-generation quarks and leptons. The interaction of the Higgs boson H with gluons[5,6] is induced in the leading order of a one-loop process, in which, with an insignificant contribution from other lighter quarks, it participates in the interaction with a virtual $t\bar{t}$-pair. Similarly, the interaction of the Higgs boson with photons is also generated through loops, although in this case the one-loop diagram with the virtual pair provides the dominant contribution [5] and destructively interferes with the smaller contribution associated with the virtual $t\bar{t}$-pair.

The discovery of the Higgs boson with a mass of $m_H \approx 125\,GeV$ at the LHC within the SM has far-reaching consequences. This fact requires paying special attention to the stability of the Higgs potential. In particular, the exact value determines the order of magnitude m_H of the self-interaction constant λ at the electroweak level and makes it possible to study its behavior up to high energy scales. A larger value m_H would mean that the self-action constant λ would become non-perturbative at some scale [7] Λ, which could be significantly lower than the Planck scale [6]. High-energy evolution of λ shows that it becomes negative at $\Lambda = O(10^{11})\,GeV$ energies. Although may λ remain positive to higher energies, perhaps up to the Planck scale if the t-quark mass exceeds its current measured value. Then the SM Higgs potential becomes unstable, and the long-term existence of the electroweak vacuum is called into question. This behavior may require new physics at an intermediate level before instability develops, that is, below M_{Planck}, although new physics at M_{Planck} could affect the stability of the electroweak vacuum and possibly change this conclusion[8].

Since the Higgs boson H^0 interacts with very heavy particles, preferably with $W^{\pm}$ and Z^0-bosons or t-quarks, and then with a lepton or b-quark, and is not directly related to massless photons or gluons, the theoretical study of the processes of formation of Higgs particles is beyond the perturbation theory (PT), both through the annihilation of an electron-positron pair into an excited Z^0-boson with further decay into and Higgs bosons H^0 become an inevitable option for studying the properties of this particle. It is also of interest to study, on the basis of ladder integral

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Bethe-Salpeter (BS) equations, for the scattering amplitude of two massive Z^0 and $W^{\pm}$- bosons, as well as t- quarks with the exchange of an infinite number of virtual Higgs particles. Research using NP methods of annihilation of electrons and positrons, as well as protons and antiprotons into an excited Z^0 boson, with further decay into Z^0- and Higgs bosons H^0 is very relevant. Theoretically, it is interesting to study beyond PT the coupling constant of the Edwards equation for the corresponding vertex functions. It is possible to describe, for example, the decay of Higgs bosons into two Z^0-bosons, or into W^+W^-.

In this work, within the framework of the standard theory $SU(2)\times U(1)$ of electroweak interaction with a R_ξ gauge with spontaneous symmetry breaking, we attempt to determine the values of the constants of three-linear and four-linear interactions of bosons ($W^{\pm}$ and Z^0) with H^0-bosons in the ladder (one-particle) approximationthr scattering amplitudes, which can be the basis for studying processes of the type $ee^+ \to Z^{0*} \to Z^0 H^0$, as well as outside the framework of TV studies of the production of the SM Higgs boson in the inclusive channel of proton-antiproton ($p\bar{p}$) collisions.

1. Bethe-Salpeter equation for scattering amplitude

It is known that among the methods for describing the scattering amplitude in the Regge-Bjorken region outside the PT framework in terms of the coupling constant, quantum field models of the BS type are physically effective. Among them, ladder models stand out, where one-particle and two-particle (bubble exchange) irreducible diagrams are summed up. The universal BS equation for the Z^0or $W^{\pm}$ bosons scattering amplitudes A with the exchange of perturbed Higgs bosons in $4D$ has the form (for graphical form see Fig.1):

$$A_{GG\to T^*\to GG}(p,p';k,k') = B(p,p') + \int d\tilde{q} K_{T^*}(q,p';k,k')\Delta_G(p-q)\Delta_G(k-q)A(q,p';k,k'), \quad (4)$$

where

$$T = H^0, \text{ or } \left(H^0H^0\right) \quad (5)$$

means single (H^0), or double $\left(H^0H^0\right)$ Higgs bosons transfers,

and

$$G = Z^0, \text{ or } W^{\pm} \quad (6)$$

bosons,

$$d\tilde{q} = \frac{d^4q}{(2\pi)^4} = \frac{dq_0|\vec{q}|^2 d|\vec{q}| d\cos\theta\, d\varphi}{(2\pi)^4}, \quad (7)$$

$$I(p,p') = V_{GGT}^2 \cdot \Delta_T(p,p') - \quad (8)$$

inhomogeneous term of equation (4),

$$K_T(q,p') = I(p,p') = V_{GGT}^2 \cdot \Delta_T(q) \; - \quad (9)$$

irreducible ladder diagrams (5) (also partially a kernel of integral equation (4)),

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$$\Delta_T(p,p') = \left[(p+p')^2 - M_T^2\right]^{-1},\ \Delta_T(q) = \left[q^2 - M_T^2\right]^{-1} - \tag{10}$$

transfers particles (in our case - Higgs bosons) propagators,

$$\Delta_G(p-q) = \left[(p-q)^2 - M_G^2\right]^{-1} - \tag{11}$$

of vector bosons propagators in the Hoft-Feynman gauge $\xi = 1$[9].
According (5), V_{GGT} - the three-particle, or four-particle vertices. In particular case V_{GGT} means for $Z^0Z^0H^0$ and $W^+W^-H^0$,

$$V_{Z^0Z^0H^0} = \frac{ig_{Z^0Z^0H^0} M_{Z^0}}{\cos\theta_W} g_{\mu\nu}\ \ ,V_{W^+W^-H^0} = ig_{W^+W^-H^0} M_{W^\pm} g_{\mu\nu}\ \ , \tag{12}$$

also, for $Z^0Z^0H^0H^0$ and $W^+W^-H^0H^0$,

$$V_{Z^0Z^0H^0H^0} = \frac{ig^2_{Z^0Z^0H^0H^0}}{2\cos^2\theta_W} g_{\mu\nu}\ \ ,V_{W^+W^-H^0H^0} = \frac{ig^2_{W^+W^-H^0H^0}}{2} g_{\mu\nu} \tag{13}$$

vertices, correspondingly.
$\cos\theta_W$ are the *cos* of the Weinberg angle[9], and $p,\ p'$ and $k,\ k',\ q$ - initial, final and transfers particles 4-moments and M_G, M_T –initial and final vector bosons masses (M_{Z^0} , or $M_{W^\pm}$) and Higgs scalars mass (M_{H^0}), correspondingly. The amplitude $A_{GG\to T^*\to GG}$ is a function of the following invariants: the total energy of the incident particles $s = (p+p')^2$, the transfers momentum $t = (p-k)^2,\ p^2,\ p'^2;\ k^2,\ k'^2$.

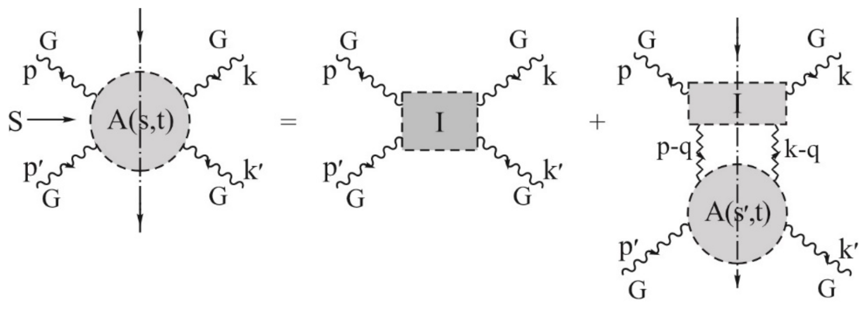

Fig.1.

Using Cutkosky's rules [10],

$$A(p,p') \to F(p,p')\theta(p_0 + p'_0) = \operatorname{Im} A(p,p'),\ 1/\left[q^2 - M_{H^0}^2\right] \to -\pi\delta\left(q^2 - M_{H^0}^2\right),$$

$$I(p,p') = \operatorname{Im} I(p,p'),\quad I(q,p') = \operatorname{Im} I(q,p') \tag{14}$$

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(in particular case $1/\left[q^2 - M^2_{H^0}\right] \to -\pi\delta\left[q^2 - M^2_{H^0}\right]$),

$$K(q, p') \to \operatorname{Im} K(q, p'), \tag{15}$$

the corresponding equation for the imaginary part of the scattering amplitude has the form

$$F(s,t) = \operatorname{Im} I(s,t) + \int d\tilde{q}\, \operatorname{Im} K_{T^*}(q, p')\Delta_G(p-q)\Delta_G(k-q)F\left(q^2, p'^2; s', t\right). \tag{16}$$

2. The case of forward scattering in one-particle transfers approximation

According to expressions (3), (5), (7), (8), (9), (10), (11), (13), (14), (15), the kernel of the equation, Feynman's rules for the propagators of vector and Higgs bosons of the vector and Higgs bosons three-linear vertex in the standard $SU(2)\times U(1)$-theory in the R_ξ-covariant gauge theory in the Hoft-Feynman gauge $\xi = 1$ [9] and Cutkosky's rules, respectively, the universal BS equation (16) for the imaginary part of Z^0 bosons forward scattering ($t = 0$) amplitude $F\left(s, p^2, p'^2\right)$ with the exchange of Higgs scalars with three-linear interactions will take the following forms,

$$F\left(s, p^2, p'^2\right) = -\pi V^2_{Z^0 Z^0 H^0}\,\delta\left(s - M^2_{H^0}\right) - \frac{\pi}{(2\pi)^4}\, V^2_{Z^0 Z^0 H^0} \int \frac{dq_0 |\vec{q}|^2 d|\vec{q}| \sin\theta\, d\theta d\varphi ds'}{\left[(p-q)^2 - M^2_{Z^0}\right]^2} \times$$

$$\delta\left(q^2 - M^2_{H^0}\right)\theta(q_0)\delta\left((p + p' - q)^2 - s'\right)\theta(p_0 + p'_0 - q_0)F\left(s', q^2, p'^2\right) \tag{17}$$

where in the integrand we include unite and $1 = \int ds'\left[(p + p' - q)^2 - s'\right]$ and in the integrand the momentum is shifted based on translational invariance $q \to p - q$ and $s' = (p + p' - q)^2$. The boundaries of integration over $\cos\theta$ and over s' are determined from the kinematic condition $\left|\cos\theta \le 1\right|$ and from the properties of the θ-function: $-1 \le \cos\theta \le 1$ and $0 \le s' \le \left(\sqrt{s} - M^2_{H^0}\right)^2$, correspondingly (at high energies $s \to \infty$ and $s >> M_{H^0}$: $0 \le s' \le s$).

2.1. The solution of equation (17)

Equation (16) have the complicating kernel and obtaining it's solution in the general case in the explicitly form is seemed to be impossible. Nevertheless analyzing the equation (16) at high energies ($s \to \infty$, $s >> M^2_{H^0}$, $M^2_{Z^0}$ one can be convinced that some terms in the kernel of equation have very small contribution to the amplitude (the imaginary part of the amplitude $F(s) = F\big|_{p^2 \to M^2_{Z^0}, p'^2 \to M^2_{Z^0}} + \Delta F\big|_{p^2 \ne M^2_{Z^0}, p'^2 \ne M^2_{Z^0}}$, the correction to the amplitude due to going beyond the mass shell at high energies is very small (of the order of $\left(p^2 - M^2_{Z^0}\right)/s$, $\left(p'^2 - M^2_{Z^0}\right)/s$) (which is quite consistent with the experimental conditions [11] $s = 14TeV$, for mass values $M_{H^0} = 125 \div 126 GeV$, $M_{Z^0} = 91.187 \pm 0.007 GeV$)

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We will solve equation (16) according of methods in [12], [13]. Integration over φ is trivial and equal to 2π . After integrating over momentum q_0 and $|\vec{q}|$ with the help of two δ -functions (further we'll suppose that $p^2 = M^2_{Z^0}$, $p'^2 = M^2_{Z^0}$ and $s \to \infty$, $s >> M^2_{H^0}$, $M^2_{Z^0}$) in the c.m.s. $\vec{p} + \vec{p}' = 0$

$$F(s) = -\frac{V^2_{Z^0 Z^0 H^0}}{128\pi^2 \sqrt{s} |\vec{p}|^2} \int_0^s ds' \left[\frac{\left(s - s' + M^2_{H^0}\right)^2}{4s} - M^2_{H^0} \right]^{-1/2} F(s') \int_{-1}^{1} \frac{d(\cos\theta)}{[a + d(\cos\theta)]^2}$$

(Here $a = \dfrac{M^2_{H^0} - \frac{p_0}{\sqrt{s}}\left(s - s' + M^2_{H^0}\right)}{2|\vec{p}| \left[\frac{\left(s - s' + M^2_{H^0}\right)^2}{4s} - M^2_{H^0} \right]^{1/2}}$) and then over $\cos\theta$ we obtain the following equation

$$F(s) = -\frac{V^2_{Z^0 Z^0 H^0}}{32\pi^2 M^2_{Z^0}} \int F(s') \left(1 - \frac{s'}{s}\right) \left[\frac{M^2_{H^0}}{M^2_{Z^0}} + \left(1 - \frac{s'}{s}\right)^2 \right]^{-1} d\left(\frac{s'}{s}\right) \quad , \tag{18}$$

i.e. having chosen a solution in the form of Regge asymptotic $F(s) = s^\alpha$ for equation (19)

$$32 \left(\frac{\pi}{g_{Z^0 Z^0 H^0}} \right)^2 = \int_0^1 d\left(\frac{s'}{s}\right) \left(\frac{s'}{s}\right)^\alpha \left(1 - \frac{s'}{s}\right) \left[1 + \left(1 - \frac{s'}{s}\right)^2 \right]^{-1} .$$

which is the sum of two hypergeometric Gaussian functions [14]

$$-64\pi^2 (\alpha + 1)(\alpha + 2) \frac{M^2_{H^0}}{V^2_{GGH^0}} = {}_2F_1\left(1,2;\alpha + 3; -i\frac{M_{Z^0}}{M_{H^0}}\right) + {}_2F_1\left(1,2;\alpha + 3; i\frac{M_{Z^0}}{M_{H^0}}\right), \tag{19}$$

Representing the hypergeometrical functions in series form am neglecting obove of leading order ($n = 0$) terms we obtain for α the following expression

$$\alpha = \frac{1}{2} \left[-3 \pm \sqrt{1 - \frac{V^2_{Z^0 Z^0 H^0}}{8\pi^2 M^2_{H^0}}} \right],$$

which makes it possible in principle to determine the values and of the interaction constants $g_{Z^0 Z^0 H^0}$. The solution of equation (16) for scattering $W^\pm$ and Z^0 bosons with the exchange of two Higgs bosons is the subject of our future elaboration[15].

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