ISSN 1512-1801

UDC 37.02

THE MODEL OF THE LEARNING PROCESS BASED ON A FUZZY APPROACH

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Abstract: *The fuzzy computer model of the teacher-knowledge-students system is proposed and its behavior under various conditions is investigated. The factors complicating the study of didactic systems is identified, the cognitive network is built that models the learning process, and its main characteristics is listed: the complexity of educational information, student's motivation, his level of understanding, class attendance, learning activity, home assignment etc. By specifying triangular membership functions, fuzzification of the input values is carried out. The computer program is written that uses a logistic function to calculate the student's average grade and the probability of receiving corresponding mark. To determine the parameters of the model, it is adjusted, during which coefficients are selected. The simulation of student learning with various input parameters is carried out. Also another model is considered, which takes into account the influence of student motivation on the amount of homework and activity in the classroom, and the results of its work are analyzed.*

Keywords: *didactics, fuzzy logic, computer, fuzzification, teaching, teacher, student.*

Introduction

In recent decades fuzzy modeling methods have become widespread [1]. They allow to create simulation models that work with fuzzy or inaccurate data, based on fuzzy logic and "soft" calculations [2; 3]. All this makes sense when studying didactic systems such as "teacher-knowledge-students" and modeling the educational process. Many indicators of the learning process are characterized by verbal descriptions and qualitative assessments: low complexity of the educational material, high student activity in the classroom, average test difficulty, etc. This makes the use of hard deterministic methods less convenient [4]. Fuzzy estimations of knowledge and skills should be used, dividing students and didactic objects into "vague" sets, and using "soft" calculations to obtain adequate conclusions [5; 6]. Losing in the accuracy and certainty of the results, fuzzy models gain in the adequacy of modeling [7; 8].

Fuzzy modeling is based on the theory of fuzzy sets developed by L. Zadeh [9]. According to it, any object, any quantity can simultaneously belong to different sets to varying degrees [6]. The truth of any statement (for example, regarding the estimation of knowledge) can take on some value in the range [0; 1]. To account for the qualitative characteristics of objects and relationships, linguistic variables are used that take values such as "low", "medium", "high", etc. Fuzzification is performed using the membership function; then soft calculations are performed, followed by defuzzification and output of modeling results in verbal form [7; 9]. All this makes it possible, by assigning qualitative characteristics of objects and relationships to linguistic variables (for example, understanding level, material complexity), to interpret judgments and obtain less accurate but more adequate conclusions.

The purpose of the research is to build a fuzzy functional model of student learning and to study its behavior when changing the didactic system parameters. **The methodological basis of the research** is the works of the following scientists: B.M. Velichkovsky [10], R. Solso [11], E.G. Skibitsky and A.G. Shabanov [12] (cognitive psychology, qualitative modeling of learning); A.P. Sviridov [13], R.V. Mayer [4; 14; 15] (mathematical and computer modeling of learning); V.V. Borisov, V.V. Kruglov and A.S. Fedulov [1], N.A. Borisov [2], A.I. Mitin and T.A. Filicheva [3], S.A. Egorov [5], V.R. Kristalinsky [6], V.N. Novikov [7], A.V. Flegontov, V.A. Duke and I.K. Fomina [8], L. Zadeh [9] (fuzzy learning modeling). Qualitative, mathematical, and computer

ISSN 1512-1801

modeling methods are used, providing for the creation of a computer program, conducting a series of computer simulations, and analyzing the results.

Discussion of the research problem

The didactic system consists of many interacting elements (for example, elements of educational material and teaching methods), interconnected by diverse connections, and is influenced by a large number of random events [10-12]. This is a poorly formalized, badly structured system [14]. Its study is complicated by the following factors: the difficulty of formalizing the university teacher's influence on the student, the vagueness and multicriteria of learning goals and objectives, the uncertainty of the characteristics of external influences, the lack of complete and accurate information about the parameters, the low predictability of student and teacher behavior, the subjectivity of expert estimations of the student's condition, the complexity of educational material. Due to the high degree of uncertainty of the components and their parameters, the use of deterministic models does not allow describing the functioning of the teacher-knowledge-students system with the required degree of adequacy [8; 9].

It follows from the principle of incompatibility, that high accuracy of measurements and predictions is incompatible with the great complexity of the system under study [9, p. 10]. Increasing the accuracy of predicting the state of a complex system leads to a decrease in the reliability of the forecast, so it is almost impossible to build a model that accurately matches the original. At the same time, according to L. Zadeh, one has to "sacrifice accuracy in the face of staggering complexity" [9, p. 10], as required by the soft systems methodology.

To create a fuzzy learning model, we will build a cognitive network of the learning process [4]. It is an oriented graph, the vertices of which are concepts corresponding to objects, factors and their characteristics, and the edges show the relations of influence (Fig. 1). In our case, the main concepts are the university teacher, the student, the educational information, the student's knowledge, class attendance, activity in the classroom, the amount of homework, testing (exam) grade or estimation. The teacher informs the students of educational information with complexity C in the amount of EI and sets homework in the amount HW. The student is characterized by the motivation M and the level of understanding LU. The student's knowledge depends on the class attendance CA, activity on lessons ACT, the homework amount done HW, etc. In the estimation process, the student's knowledge level LK is determined using a test. Its difficulty is set by the teacher; he also gives a grade G.

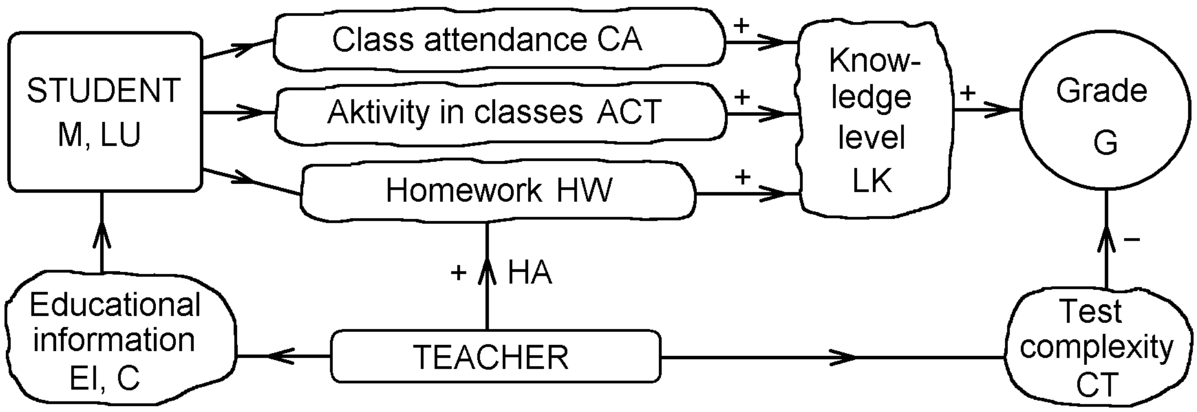

Fig. 1. Cognitive network of the learning process.

The prerequisites (parameters, input values) of a fuzzy functional learning model are represented as fuzzy sets, and the output values (grade and/or mark probabilities) are calculated using functions. An important step is fuzzification (the introduction of fuzziness), that is, the determination of the values of the membership function of terms (fuzzy sets) based on verbal estimates of input quantities.

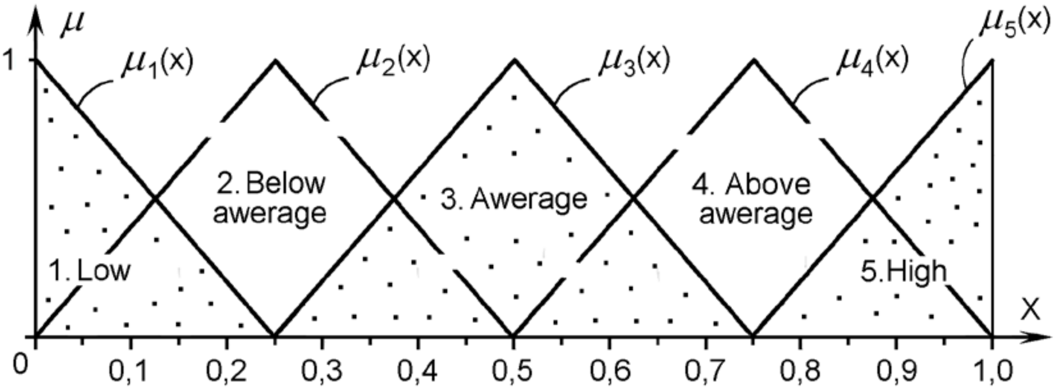

Fig. 2. Membership function used for fuzzification.

The linguistic variables "level of understanding" $LU_{Л}$, "complexity of the educational material" C_L, "activity in the classroom" ACT_L, "amount of homework (including preparation for tests and exams)" HW_L, "complexity of the test" CT_L correspond to the term set {low, below average, average, above average, high}. Let's match it with a set of numerical values and introduce membership functions showing to what extent the value of x belongs to the category k (Fig. 2).

The results of the research

Consider a discipline that is studied 1 couple (1.5 hours) per week. If a student works on homework for about 60 minutes per week, then this corresponds to HW_L = "high", 30 minutes per week – HW_L = "medium", less than 10 minutes per week – HW_L = "low". The variables LU_L, C_L, AKT_L, HW_L and CT_L take 5 values (Fig. 2): k=1 – low, $x \in [0; 0{,}25]$; k=2 – below average $x \in [0; 0{,}5]$; k=3 – average $x \in [0{,}25; 0{,}75]$; k=4 – above average $x \in [0{,}5;1]$; k=5 – high $x \in [0{,}75;1]$. The attendance of classes CA is equal to the proportion of classes attended by the student ($CA \in [0;1]$). To assess academic performance, we will introduce the linguistic variable G_L = {no knowledge, unsatisfactory, satisfactory, good, excellent}.

Need to construct a function linking the knowledge level LK (the ratio of the amount of knowledge acquired by the student to the amount of information reported) with the factors listed above. Let's take the sigmoidal (logistic) function, varying from 0 to 1, as a basis:

$$LK = \frac{1}{1 + \exp(a - LU \cdot [b \cdot CA \cdot ACT + c \cdot CA \cdot HW + d \cdot HW] / CT)}, \quad G = \frac{LK}{CT},$$

where a, b, c, and d are the parameters determined during the model configuration process. It is taken into account that the level of knowledge LK is growing with an increase in: 1) the student attendance, student activity in the class and the amount of homework, etc.; 2) the fraction LU/CT, which shows how many times the level of understanding is greater than the complexity of the test. The product of CA*ACT and CA*HW allow us to take into account the synergy of these factors. The estimation of student's knowledge is calculated as follows: G = LK/CT.

The applied program on *ABCPascal* contains the function ZNACH(k) for finding numerical values of C, LU, ACT, HW and CT, based on the input values of the verbal variable: "high" – k = 5, ..., "medium" – k = 3, ... etc. In the *Repeat ... until ...;* cycle, the grade G_i (i = 1, ..., N) and its average value G are calculated repeatedly (N =10000), and the probabilities of marks are found: p(excellent) = p_5, p(good) = p_4, ..., p(no knowledge) = p_1 as the ratio of the number of their "getting" to N. Defuzzification is carried out as follows: if G > 0.9, then G_L = "excellent"; if $0{,}7 < G \leq 0{,}9$, then G_L = "good"; if $0.5 < G \leq 0.7$, then G_L = "satisfactory"; if $0.2 < G \leq 0.5$, then G_L = "unsatisfactory" ; if $G \leq 0.2$, then G_L = "no knowledge".

In simulation modeling, the correct choice of model parameters is of great importance. A computer program that simulates the functioning of the "teacher-knowledge-students" system actually calculates a function of many variables – the knowledge level of the students or the score for the test, depending on five parameters of the educational process. The compliance requirement

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must be met: these parameters are selected so that, with a given (reasonable) organization of training, the simulation results correspond to the real values obtained during testing or pedagogical observation [14; 15].

Program 1. Fuzzy learning model (ABCPascal)

```
uses crt,graphABC; const N=10000;
var i,j,k,x1,x2,x3,x4,x5: integer;
SL,UP,PZ,ACT,DR,DZ,CT,O,UZ,S,UZN,M,x,y,z,a:single;
Function ZNACH(k:integer):single; Label m;
begin m: x:=random(1000)/1000; If k=1 then a:=0;
If k=2 then a:=0.25; If k=3 then a:=0.5;
If k=4 then a:=0.75; If k=5 then a:=1;
If x<=a then y:=4*x-k+2; If x>=a then y:=k-4*x;
z:=random(1000)/1000; If z<y then ZNACH:=x else goto m; end;
BEGIN Randomize;
Repeat inc(i); SL:=ZNACH(3); UP:=ZNACH(4);
  {DZ:=ZNACH(3); M:=ZNACH(3); ACT:=M; DR:=1-exp(-2*M*DZ);}
  PZ:=0.8; ACT:=ZNACH(3); DR:=ZNACH(3); {ACT:=1; DR:=1;}
  UZ:=1/(1+exp(3-UP/SL*(1.73*PZ*ACT+2*PZ*DR+1.28*DR)));
  CT:=ZNACH(3); O:=(UZ/CT);
  If O>1 then O:=1; If O>0.9 then inc(x5);
  If (O>0.7)and(O<=0.9) then inc(x4);
  If (O>0.5)and(O<=0.7) then inc(x3);
  If (O>0.2)and(O<=0.5) then inc(x2);
  If O<=0.2 then inc(x1); S:=S+O;
until i=N; writeln('G = ',S/N);
writeln('no knowledge ',x1/N); writeln('unsatisf.',x2/N);
writeln('satisfact.',x3/N); writeln('good ',x4/N);
writeln('excellent',x5/N); END.
```

To set the model parameters, it is necessary to configure it. In this case, we will proceed from the expected values of G', presented in the table. 1. It considers the following limiting cases: 1) the student does not study (variables CA, ACT, etc. are assigned zero values), score G' = 0.1; 2) the student does not attend classes, working at home for 1.5 hours a week (CA = ACT = 0, HW = 1), score G' = 0.35; 3) attendance and activity high, the student does not work at home (CA = ACT = 1, HW = 0), grade G' = 0.5; 4) attendance, activity in the classroom, the amount of homework is high (CA = ACT = HW = 1), CT_L = "average", grade G' = 1. In all cases, $LU_L = C_L = CT_L$ = "average". The last column contains the results of calculations G after configuring the model.

Table 1. Input and output values for configuring the model.

N	C	LU	CA	ACT	HW	CT	G'	G
1	average	average	0	0	0	average	0,1	0,1
2	average	average	0	0	1	average	0,35	0,35
3	average	average	1	1	0	average	0,5	0,5
4	average	average	1	1	1	average	1	0,99

Substituting the values from the first row of the table, we select coefficient a so that G = 0.1. It turns out: a = 3. Then we substitute the values from the second row of the table and select coefficient d so that G = 0.35. It turns out: d = 1.28. Similarly, b and c are found. It turns out:

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$$G = \frac{1}{1 + \exp(3 - LU \cdot [1{,}73\, CA \cdot ACT + 2\, CA \cdot HW + 1{,}28\, HW] / CT)}.$$

The model allows for a series of computational experiments:

1) difficulty C_L = "average", level of understanding LU_L = "above average", attendance CA = 0.8, activity AKT_L = "average", homework HW_L = "average", complexity of the test CT_L = "average"; the probabilities of grades: p(no knowledge)=0, p(unsatisfactory)=0.05, p(satisfactory) = 0.12, p(good)=0.16, p(excellent)=0.67, average grade G = 0.89.

2) increase the complexity of the material (C_L= "above average"), leave everything else unchanged; then the probabilities of estimates: p(no knowledge)=0, p(unsatisfactory) = 0.34, p(satisfactory) = 0.31, p(good) = 0.18, p(excellent) = 0.17, average grade G = 0.63.

3) increase class attendance: CA = 1, leave everything else as in the first case; the probabilities of estimates: p(no knowledge) = 0, p(unsatisfactory) = 0.01, p(satisfactory) = 0.06, p(good) = 0.10, p(excellent) = 0.83, average grade G = 0.95.

4) increase activity in class: AKT_L = "above average", everything else is as in the first case; then the probabilities of estimates: p(no knowledge) = 0, p(unsatisfactory) = 0.01, p(satisfactory) = 0.06, p(good) = 0.10, p(excellent) = 0.83, average grade G = 0.95.

5) reduce amount of homework: HW_L = "below average", leave everything else as in the first case; the probabilities of estimates: p(no knowledge) = 0.01, p(unsatisfactory) = 0.37, p(satisfactory) = 0.23, p(good) = 0.15, p(excellent) = 0.23, average grade G = 0.62.

6) increase the complexity of the test: CT_L = "above average", everything else is as in the first case; probabilities of estimates: p(no knowledge) = 0, p(unsatisfactory) = 0.21, p(satisfactory) = 0.26, p(good) = 0.23, p(excellent) = 0.3, average grade G = 0.72.

Thus, the model reacts correctly to changes in the parameters of the learning process. The dependence of the score G on the student's activity in the class ACT and the amount of homework HW is shown in Fig. 3. The model has 6 input values, each of which, with the exception of CA, can take five values. With a fixed class attendance $5^5 = 3125$ different combinations are obtained.

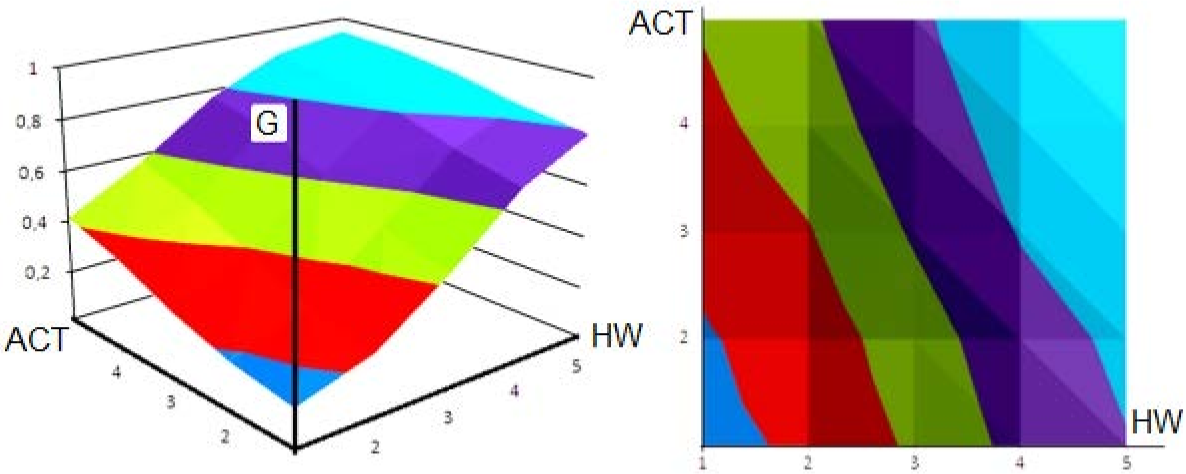

Fig. 3. The dependence of the student's knowledge estimation on the ACT and HW.

Let's make so that the model takes into account: 1) the influence of motivation on the student; 2) the amount of homework given by the teacher. The student's motivation has a positive effect on the activity of work in the classroom (ACT = M) and the amount of homework HW, which also depends on the home assignment HA. It may include preparing for a lesson, for a test or exam, writing an essay, etc. Let's assume that $HW \approx 1 - \exp(-\alpha \cdot M \cdot HA)$, where $\alpha -$ the certain coefficient (since M, HA, and HW vary in the range [0; 1], then $\alpha \approx 2$). Then, with $C_L = LU_L = M_L = HW_L = CT_L$ = "average" and CA = 0.8, we get: probabilities of verbal marks: p(no knowledge) = 0.03, p(unsatisfactory) = 0.51, p(satisfactory) = 0.21, p(good) = 0.11, p(excellent) = 0.13, the score G is about 0.53.

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Table 2. Some simulation results (CA=0.8).

M	HW	G	p_1	p_2	p_3	p_4	p_5
average	low	0,25	0,39	0,56	0,03	0,01	0,00
high	low	0,47	0,04	0,60	0,20	0,09	0,08
high	average	0,84	0	0,1	0,16	0,18	0,57
high	high	0,93	0	0,03	0,08	0,12	0,78

If you increase the home assignment HA_L to an "above average" value, the learning result improves: the probabilities of grades: p(no knowledge) = 0.01, p(unsatisfactory) = 0.34, p(satisfactory) = 0.23, p(good) = 0.16, p(excellent) = 0.26, average grade G = 0.65. Now we will increase motivation to M_L = "above average", leaving the HW_L and other parameters at the "average" level. It turns out: p(no knowledge) = 0, p(unsatisfactory) = 0.2, p(satisfactory) = 0.21, p(good) = 0.18, p(excellent) = 0.4, grade G = 0.75. The model takes into account the decisive influence of student motivation on learning outcomes; if M_L = "below average", then with C_L = LU_L = HW_L = CT_L = "average" it turns out: p(no knowledge) = 0.38, p(unsatisfactory) = 0.53, p(satisfactory) = 0.06, p(good) = 0.02, p(excellent) = 0, the estimate is G = 0.28. Table 2 shows the simulation results for C_L = LU_L = CT_L = "average" and CA = 0.8.

Conclusions

The computer program is proposed that performs fuzzy modeling of learning, taking into account the complexity of the educational material, the student's level of understanding, his motivation, class attendance, activity in the classroom, the amount of home assignment and homework, and the teacher's requirements for testing. The model is configured by selecting its parameters, and the results of its use in modeling the learning process are presented. At the output, the model provides a numerical estimation of the student's knowledge and the probability of getting marks. The proposed model allows us to take into account the influence of student motivation on activity in the classroom and the amount of homework. It helps to predict learning outcomes; more complex models can be built on its basis.

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В статье 3 рисунка и 2 таблицы.

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Article received 2025-05-05