Georgian Electronic Scientific Journals: Physics \#1(38-1)-2003

# DIFFRACTION RADIATION OF AN ELECTRON FLOW MOVING ABOVE THE SCREEN WITH SLOTS 

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Accept for publication May,2002
ABSTRACT. The problem of diffraction of an internal field of moving electron flow on a perforated screen of final width is being solved. The quantitative theory of phenomenon of diffraction radiation is constructed and physical interpretation is given on the basis of numerical results.

Let us assume that parallel with surface of a boundless perforated screen (Fig.1) the monochromatic electronic flow with constant speed is moving and instantaneous value of a density of charge is the following:

$$
\begin{equation*}
\rho=\rho_{0} \delta(\mathrm{z}-\varsigma) \exp \left[\operatorname{ih}\left(\alpha_{1} \mathrm{x}+\alpha_{2} \mathrm{y}\right)-\omega \mathrm{t}\right] \tag{1}
\end{equation*}
$$



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where $\rho_{0}$ is amplitude modulation of electronic modulation; $\mathrm{h}=\mathrm{k} / \beta ; \mathrm{k}=\omega / \mathrm{c} ; \beta=\mathrm{v} / \mathrm{c}$ relative v

Fig. 1 equency of low; c free space velocity; $\mathrm{v}=\mathrm{v}\left(\mathrm{x}_{0} \alpha_{1}+\mathrm{y}_{0} \alpha_{2}\right) ; \delta(\mathrm{z}-\zeta)$ is Dirace function; $\alpha_{1}, \alpha_{2}$ are direction cosine $\left(\alpha_{1}^{2}+\alpha_{2}^{2}=1\right) ; \zeta$ is aiming distance; $x_{0}, y_{0}$ basis vector of coordinat system.

Field source represented by (1) is a real model for investigation of initiated diffraction radiation [1].

The sought field in a space is represented as a superposition of falling and reflected waves of concrete sectors, as a double Fourier line with certain unknown coefficients (The field over the screen $\mathrm{Z}>0$ ) is represented as a superposition of internal field of charges and field, scattered on a perforated screen):

$$
\begin{align*}
& \left\{\begin{array}{l}
\mathrm{H}_{\mathrm{i}}^{+}=\mathrm{H}_{\mathrm{i}}^{0}+\Lambda \sum_{\mathrm{m}, \mathrm{n}=-\infty}^{\infty} \mathrm{A}_{\mathrm{mn}}^{\mathrm{i}} \exp \psi(\mathrm{x}, \mathrm{y}, \mathrm{z}) \\
\mathrm{E}_{\mathrm{i}}^{+}=\mathrm{E}_{\mathrm{i}}^{0}-\Lambda \sum_{\mathrm{m}, \mathrm{n}=-\infty}^{\infty} \mathrm{B}_{\mathrm{mn}}^{\mathrm{i}} \exp \psi(\mathrm{x}, \mathrm{y}, \mathrm{z})
\end{array}\right. \\
& \text { ( } \mathrm{Z}>0 \text { ) }  \tag{2}\\
& \left\{\begin{array}{l}
\mathrm{H}_{\mathrm{i}}^{-}=\Lambda \sum_{\mathrm{m}, \mathrm{n}=-\infty}^{\infty} \widetilde{\mathrm{A}}_{\mathrm{mn}}^{\mathrm{i}} \exp \varphi(\mathrm{x}, \mathrm{y}, \mathrm{z}) ; \\
\mathrm{E}_{\mathrm{i}}^{-}=\Lambda \sum_{\mathrm{m}, \mathrm{n}=-\infty}^{\infty} \widetilde{\mathrm{B}}_{\mathrm{mn}}^{\mathrm{i}} \exp \varphi(\mathrm{x}, \mathrm{y}, \mathrm{z}) ;
\end{array}\right. \\
& \text { ( } \mathrm{Z}<-\mathrm{S} \text { ) } \\
& \left\{\begin{array}{l}
H_{i}^{\text {ins }}=\Lambda \sum_{j=1}^{2} \sum_{p, q=0}^{\infty} \Delta_{i}^{j}\left(X_{p, q}^{j} e^{\gamma^{j}}, Y_{p, q}^{j} e^{-\gamma^{j}}\right) \Phi_{p, q}^{i}\left(r_{o}, x, y\right) ; \\
E_{i}^{\text {ins }}=\Lambda \sum_{j=1}^{2} \sum_{p, q=0}^{\infty} \Theta_{i}^{j}\left(X_{p, q}^{j} e^{\gamma^{j} z}, Y_{p, q}^{j} e^{-\gamma^{j}}\right) F_{p, q}^{i}\left(r_{o}, x, y\right) ;
\end{array}\right.  \tag{3}\\
& \text { ( } 0 \geq \mathrm{Z} \geq-\mathrm{S} \text { ). }
\end{align*}
$$

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Here $\mathrm{I}=\mathrm{x}, \mathrm{y}, \mathrm{z} ; \quad \psi_{\mathrm{mn}}=\mathrm{i}\left(\mathrm{h}_{\mathrm{m}} \mathrm{x}+\mathrm{h}_{\mathrm{n}} \mathrm{y}+\mathrm{h}_{\mathrm{mn}} \mathrm{z}\right) ; \quad \varphi_{\mathrm{mn}}=\mathrm{i}\left(\mathrm{h}_{\mathrm{m}} \mathrm{x}+\right.$ $\left.+\mathrm{h}_{\mathrm{n}} \mathrm{y}-\mathrm{h}_{\mathrm{mn}}(\mathrm{z}+\mathrm{s})\right) ; \quad \mathrm{h}_{\mathrm{m}}=\mathrm{h} \alpha_{1}+2 \pi \mathrm{~m} / \mathrm{d} ; \quad \mathrm{h}_{\mathrm{n}}=\mathrm{h} \alpha_{2}+2 \pi \mathrm{n} / \mathrm{l}$; $\mathrm{h}_{\mathrm{mn}}=\sqrt{\mathrm{k}^{2}-\mathrm{h}_{\mathrm{m}}{ }^{2}-\mathrm{h}_{\mathrm{n}}{ }^{2}} ;$ the constant $\quad \Lambda=\left(\rho_{0} \mathrm{c} / 2\right) \sqrt{1-\beta^{2}} \times$ $\times \exp \left[-(\mathrm{k} \zeta / \beta) \sqrt{1-\beta^{2}}\right]$ entered as a matter of convenience calculations; $\quad \gamma^{\mathrm{j}}=\sqrt{\mathrm{k}_{\mathrm{pqj}}^{2}-\varepsilon_{\mathrm{r}} \mathrm{k}^{2}} ; \quad \varepsilon_{\mathrm{r}} \quad$ a relative permittivity of environment inside holes; $\mathrm{k}_{\mathrm{pqj}}$ eigenvalues; and $\Phi_{\mathrm{p}, \mathrm{q}}^{\mathrm{j}}\left(\mathrm{r}_{\mathrm{o}}, \mathrm{x}, \mathrm{y}\right)$, $\mathrm{F}_{\mathrm{p}, \mathrm{q}}^{\mathrm{j}}\left(\mathrm{r}_{\mathrm{o}}, \mathrm{x}, \mathrm{y}\right)$ fundamental functions of wave guides correspond of the configuration $\mathbf{r}_{0}$ is a geometrical size (the index $\mathrm{j}=1$ corresponds to waves $E$ of a type, and $j=2$ to waves $H$ of a type); $E^{0}$ and $H^{0}$ vectors of an internal field of moving charges; $\mathrm{A}_{\mathrm{mn}}^{\mathrm{i}}, \widetilde{\mathrm{A}}_{\mathrm{mn}}^{\mathrm{i}}, \mathrm{B}_{\mathrm{mn}}^{\mathrm{i}}, \widetilde{\mathrm{B}}_{\mathrm{mn}}^{\mathrm{i}}, \mathrm{X}_{\mathrm{pq}}^{\mathrm{j}}, \mathrm{Y}_{\mathrm{pq}}^{\mathrm{j}}$ unknown coefficients which are being a subject of definition.

Satisfying boundary conditions on a surface of metal (planes z $=0$ and $\mathrm{z}=-\mathrm{S}$ ), leads to the functional equations, which may be transformed, by means of moment method, to the following infinite linear algebraic systems of pair equations.

$$
\left\{\begin{array}{l}
\lambda_{v \mu} Z_{v \mu}^{ \pm}+G_{v \mu} t_{v \mu}^{ \pm}-r_{v \mu} \sum_{m, n=-\infty}^{\infty}\left(D_{m n}^{(1)} t_{m n}^{ \pm}+D_{m n}^{(2)} Z_{m n}^{ \pm}\right)=V_{v \mu}^{(1)}  \tag{4}\\
P_{v \mu} Z_{v \mu}^{ \pm}+\lambda_{v \mu} t_{v \mu}^{ \pm}-r_{v \mu} \sum_{m, n=-\infty}^{\infty}\left(D_{m n}^{(3)} t_{m n}^{ \pm}+D_{m n}^{(4)} Z_{m n}^{ \pm}\right)=V_{v \mu}^{(2)}
\end{array}\right.
$$

where

$$
\mathrm{Z}_{\mathrm{mn}}^{ \pm}=\mathrm{A}_{\mathrm{mn}}^{\mathrm{x}} \pm \widetilde{\mathrm{A}}_{\mathrm{mn}}^{\mathrm{x}}+\frac{\alpha_{1} \beta}{\sqrt{1-\beta^{2}}} \delta_{\mathrm{m} 0} \delta_{\mathrm{n} 0}
$$

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$$
\begin{gathered}
\mathrm{t}_{\mathrm{mn}}^{ \pm}=\mathrm{A}_{\mathrm{mn}}^{\mathrm{y}} \pm \widetilde{\mathrm{A}}_{\mathrm{mn}}^{\mathrm{y}}-\frac{\alpha_{2} \beta}{\sqrt{1-\beta^{2}}} \delta_{\mathrm{m} 0} \delta_{\mathrm{n} 0} ; \\
\mathrm{V}_{\mathrm{v} \mu}^{(1)}=-\frac{2 \alpha_{1}}{\beta} \sqrt{1-\beta^{2}} \delta_{v 0} \delta_{\mu 0}, \mathrm{~V}_{\mathrm{v} \mathrm{\mu}}^{(2)}=\frac{2 \alpha_{2}}{\beta} \sqrt{1-\beta^{2}} \delta_{\mathrm{v} 0} \delta_{\mu 0},
\end{gathered}
$$

the remaining values are identical to expressions conducted in [2-5].
Having conducted research of matrix elements and free members (4) and having convinced in quadratic convergence in space of Hilbert $1^{2}$ we conclude that the equations are Fredholm type and are solved by method of a reduction.

Determining unknown coefficients of a problem, we build relation of a flow of power of a spatial harmonic from a single site of a screen $\left(\Pi_{\mathrm{mn}}\right)$ from miscellaneous geometrical and electrical parameters.

$$
\begin{equation*}
\Pi_{\mathrm{mn}}=\frac{\mathrm{J}_{0}^{2}\left(1-\beta^{2}\right)}{4 \beta^{2}} \exp \left[-\frac{4 \pi \zeta \sqrt{1-\beta^{2}}}{\beta \lambda}\right] \cdot \mathrm{Z}_{0} \Omega_{\mathrm{mn}}=\frac{\mathrm{J}_{0}^{2}}{2} \mathrm{R}_{\Sigma} \tag{5}
\end{equation*}
$$

where $\mathrm{J}_{0}$ is a line density of a current; $\mathrm{Z}_{0}$ an environmental wave resistance;

$$
\begin{aligned}
\Omega_{\mathrm{mn}}=\left(1+\frac{\mathrm{h}_{\mathrm{m}}^{2}}{\mathrm{~h}_{\mathrm{mn}}^{2}}\right) \cdot\left|\mathrm{A}_{\mathrm{mn}}^{\mathrm{x}}\right|^{2}+\left(1+\frac{\mathrm{h}_{\mathrm{n}}^{2}}{\mathrm{~h}_{\mathrm{mn}}^{2}}\right) \cdot\left|\mathrm{A}_{\mathrm{mn}}^{\mathrm{y}}\right|^{2} & + \\
& +2 \frac{\mathrm{~h}_{\mathrm{m}} \mathrm{~h}_{\mathrm{n}}}{\mathrm{~h}_{\mathrm{mn}}^{2}} \operatorname{Re}\left(\mathrm{~A}_{\mathrm{mn}}^{\mathrm{x}} \mathrm{~A}_{\mathrm{mn}}^{\mathrm{y}^{*}}\right)
\end{aligned}
$$

(* - complex conjugate value).
Numerical computation was carried out for rectangular slots. On the basis of these numerical data the analysis of physical features of diffraction radiation is conducted.

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The Fig. 2 illustrates exponential decrease of a radiation energy with increase of the distance between electronic stream and lattice. As it is visible from a figure, starting from the value $\zeta=\zeta_{0} / \lambda=0.1$ the radiation becomes practically unobservable.

In Fig. 3 the dependence of $R_{\Sigma}$ on the speed of the beam $\beta=v / c$ is presented. The continuous line defines areas of meaning, where the condition of radiation is carried out for the data $\mathrm{m}, \mathrm{n}, \chi$. The dotted line defines areas, where this condition is not carried out. In all cases


## Fig. 2

the diffraction radiation disappears, when the speed of a flow approaches to the speed of light. As is known [1] currents inducing radiation undergo Lorenze reduction and aspires to zero at $\beta \rightarrow 1$.

From Figs. 4-5 where the dependence of factor $\mathrm{R}_{\Sigma}$ as a function of geometrical parameters of a lattice $\delta$ and $\theta$ is presented obvious, that the radiating power is periodic function of the depth of lattice. The characteristic resonance takes place, when the depth of lattice is approximately multiple $\mathrm{k} / 2 \gamma^{\mathrm{j}}$.

In Fig. 6 the dependence $\mathrm{R}_{\Sigma}$ on parameter $\chi$ (frequency of fluctuation) at the fixed depth of the screen and various meanings of factor of filling is given. The substantial growth of capacity of

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radiation as a result of a choice of appropriate depth of lattice is appreciable only at width of slots, smaller than half-cycle of structure. The sharp change of intensity of radiation in a neighborhood of meanings is observed

$$
\chi^{+}=\frac{\mid \mathrm{m} \beta}{\sqrt{\beta^{2}-\left(\alpha_{2}+\mathrm{n} \beta / \chi_{2}\right)^{2}-\alpha_{1}}} ; \chi^{-}=\frac{\mid \mathrm{m} \beta}{\sqrt{\beta^{2}-\left(\alpha_{2}+\mathrm{n} \beta / \chi_{2}\right)^{2}+\alpha_{1}}} ; \text { (6) }
$$

( $\mathrm{m}=-1,-2, \ldots \mathrm{n}= \pm 1, \pm 2, \ldots$ ) which is connected to the resonant phenomena similar to Wood anomalies. The formula (6) precisely predicts conditions, at which the Wood anomalies in diffraction radiation take place.


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Fig. 3


Fig. 4

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Fig. 5


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Fig. 6

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