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CONTINUOUS ADAPTATION OF THRESHOLD BODIES ON ALGORITHM OF THE UNIFIED ENCOURAGEMENT AND INDIVIDUAL PUNISHMENT (UEIP)

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ABSTRACT. The theory of continuous adaptation without a feedback and with a feedback on algorithm of the unified encouragement and individual punishment (UEIP) is stated at threshold reservation of the binary channels used for transfer of the information, received in physical experiments. The results of machine modelling of behaviour of threshold decision body are given during adaptation on algorithm UEIP.

During continuous adaptation without a feedback on Widrow-Hoff algorithm [1] on each step of iteration the size of an increment of weights is determined by a difference between the random weighted sum of input signals and restriction overlapped on it. However other algorithms of an increment of weights may use direct comparison of a signal $\mathrm{X_i}$ on *i*-th input of threshold body with a right answer submitted from the outside X (or the decision Y in the presence of a feedback). For example, it is possible to start with a principle of encouragement and punishment, according to which changes in a vector of weights (including a threshold as one of the components) are made on each step of iteration, however in the absence of a mistake weight is increased by the size dependent on number of iteration, at detection of a mistake it decreases on the size dependent both on iteration number and value of weight. Thus, an increment of *i*-th weight

$$\Delta a_{\mathrm{i}}(k) = a_{\mathrm{i}}(k+1) - a_{\mathrm{i}}(k), \tag{1}$$

made on (k+1)-th step of iteration, in the absence of a mistake accepts some value β_k with the probability $1-q_i$, and in the presence of a mistake – the value $\beta_k \cdot e^{a_i(k)}$ with probability q_i:

$$\Delta a_i(k)=\begin{cases} \beta_k \text{ with probability } 1-q_i \\ -\beta_k \cdot e^{a_i(k)} \text{ with probability } q_i \end{cases}, \tag{2}$$

where $\beta_k \geq 0$. The initial weights $a_i(1)\left(i=\overline{1, n+1}\right)$ can be given arbitrary. Thus, in considered strategy the encouragement is unified, it does not depend on an input of threshold body and is determined only by the number of iteration and punishment is applied strictly individually. This strategy can be named algorithm of the unified encouragement and individual punishment (UEIP).

Expectation value of $\Delta a_i(k)$ has the following form:

$$M[\Delta a_i(k)]=\beta_k(1-q_i)-\beta_k e^{a_i(k)}q_i=\beta_k q_i\left(\frac{1-q_i}{q_i}-e^{a_i(k)}\right). \tag{3}$$

Let in the established condition

$$\left.\begin{array}{c} M[\Delta a_i(k)]=0 \\ i=\overline{1,n+1} \end{array}\right\}. \tag{4}$$

As before, the values of weights at which such condition is reached, we designate as $\hat{a}_i$. Then for $\hat{a}_i\left(i=\overline{1,n+1}\right)$ the following equations are obtained

$$\left.\begin{array}{c} \hat{a}_i=\ln\dfrac{1-q_i}{q_i} \\ i=\overline{1,n+1} \end{array}\right\}. \tag{5}$$

Thus, during continuous adaptation without a feedback by algorithm (2) the increments of weights, the latter are established at the levels coordinated with criterion of a maximum a posteriori probability if only the ratios (4) take place. Particularly this condition will be fulfilled, if the value β_k, corresponding $(k+1)-th$ step of iteration is determined by the formula

$$\beta_k = \frac{1}{k}. \tag{6}$$

It is possible to prove, that such choice is in agreement with the one developed in [2] Bayesian approach giving for a statistical estimation of weight the value $\ln\frac{1-q_i}{q_i}$

$$a_i = \ln\frac{M - n_i + 1}{n_i + 1}, \tag{7}$$

where M is the number of comparisons of a signal X_i on *i*-th input of decision body with a right answer submitted from the outside X, and n_i is the amount of mistakes observed at it. It is supposed, that a priori distribution of a random value $\hat{q}_i = N_i/M$ is uniform.

Really, let

$$a_i(k) = \ln\frac{k - n_i + 1}{n_i + 1}. \tag{8}$$

There is an estimation of value $\ln\frac{1-q_i}{q_i}$ on $k-th$ step of adaptation, i.e. by results of k comparisons of a signal X_i with a right answer submitted from the outside X, at the number of observed mistakes equal to n. From here it follows

$$\frac{k - n_i + 1}{n_i + 1} = e^{a_i(k)}, \qquad (9)$$

$$\frac{1}{n_i + 1} = \frac{e^{a_i(k)}}{k - n_i + 1}. \qquad (10)$$

Then on the following step, depending on result of comparison, we have

$$a_i(k+1) = \ln\left(\frac{(k+1) - n_i + 1}{n_i + 1}\right) = \ln\left(\frac{k - n_i + 1}{n_i + 1} + \frac{1}{n_i + 1}\right). \qquad (11)$$

at concurrence of signals and

$$a_i(k+1) = \ln\left(\frac{(k+1) - (n_i + 1) + 1}{(n_i + 1) + 1}\right) = \ln\left(\frac{k - n_i + 1}{n_i + 1}\right) - \ln\left(1 + \frac{1}{n_i + 1}\right) \qquad (12)$$

otherwise.

Taking into account (9) and (10) in last ratios, we get:

$$a_i(k+1) = \begin{cases} a_i(k) + \ln\left(1 + \dfrac{1}{k - n_i + 1}\right) & \text{with probability } 1 - q_i \\ a_i(k) - \ln\left(1 + \dfrac{e^{a_i(k)}}{k - n_i + 1}\right) & \text{with probability } q_i \end{cases}. \qquad (13)$$

At $k >> n_i >> 1$, using valid at $x \to 0$ ratio $\ln(1 + x) = x$, we get:

$$a_i(k+1) = \begin{cases} a_i(k) + \dfrac{1}{k} & \text{with probability } 1 - q_i \\ a_i(k) - \dfrac{1}{k} \cdot e^{a_i(k)} & \text{with probability } q_i \end{cases}. \qquad (13')$$

Introducing the designation (6), we come to the former algorithm (2) of increment weights, which proves our statement.

Thus, in a first approximation the algorithm (2) is equivalent to estimation of weights $\ln\frac{1-q_i}{q_i}$ on a ratio $\ln\frac{k-n_i+1}{n_i+1}$ on each step of iteration. Therefore there exists (11) value M determined by the formula of number of k iterations, at which statistical estimation of $\hat{q}_i$ probabilities of a mistake q_i deviates from this probability q_i not more, than on the given small size ε with probability close enough to unit α. Due to the fact that the ratio $\frac{1}{k}\geq\frac{1}{M}$ takes place before the achievement of this condition at $k \leq M$, the use of smaller value of a constant increment of weights $\beta=\frac{1}{M}$ instead of β_k during adaptation, cannot break convergence of the process to the established condition:

$$\beta_k \equiv \beta = \frac{1}{M}. \tag{14}$$

At continuous adaptation with a feedback on algorithm of the unified encouragement and individual punishment the established values of weights are determined by formulas

$$\left.\begin{aligned} \hat{a}_i &= \ln\frac{1-d_i}{d_i} \\ i &= \overline{1,n+1} \end{aligned}\right\}, \tag{15}$$

where d_i is the probability of a mismatch of a signal X_i on $i-th$ input of threshold body with the decision Y on its output. For an estimation of deviations of these weights from values $\ln\frac{1-q_i}{q_i}$ it is necessary to start with ratio

$$\left.\begin{array}{c} q_i - Q \le d_i \le q_i + Q \\ i = \overline{1, n+1} \end{array}\right\},$$

where Q is the probability of a mistake of threshold body.

The process of random wandering of weights at continuous adaptation causes the appropriate statistical distribution of probability of a mistake of Q threshold element which can be considered as a function of external entrance parameters $q_1, q_2, \ldots, q_n, q_{n+1}$ and internal entrance parameters $a_1, a_2, \ldots, a_n, a_{n+1}$.

As the dependence

$$Q = Q(a_1, a_2, \ldots, a_n, a_{n+1}, q_1, q_2, \ldots, q_n, q_{n+1})$$

is known to us algorithmically [3] and as an appropriate program, it is more convenient to study statistical distribution Q using Monte Carlo method [4], treating the characteristic Q of threshold body as a random value Q^*, being the function of random arguments $a_1, a_2, \ldots, a_n, a_{n+1}$ and fixed parameters $q_i\left(i = \overline{1, n+1}\right)$. Realizations of this random value are designated by a symbol $\hat{Q}\left(0 \le \hat{Q} \le 1\right)$.

Knowledge of the histogram or density $f_Q\left(\hat{Q}\right)$ of distribution of a random value Q^* allows to calculate the probability that Q^* is less than some maximally admited value Q_0:

$$P_0 = \Pr\left\{Q^* < Q_0\right\}$$

Taking close to unit value of this probability, it is possible to find the value β of the weight increments, providing the required probability P_0.

For machine modelling of threshold body behaviour during adaptation and the circle of questions outlined above we created an appropriate program. It allows to study adaptation on algorithm of the unified encouragement and individual punishment at presence of a feedback. Its input data are:

- The activator of the generator of the random numbers evenly distributed in an interval (0, 1);
- The amount of channels of threshold body (not more than 11);
- Number of the channel, which weight distribution of during the adaptation is desirable to receive;
- Number of steps of adaptation (no more than ten thousand);
- The value β, serving as a measure of an increment of weights on each step of adaptation;
- Probabilities of refusal of channels and their arbitraly given initial weights (by one card on each channel);
- Aprioristic probability of presentation to threshold body for recognition of a signal $+1$ and initial value of a threshold.

The output information contains all set of the input data, and also distributions of weight interesting for the user of the channel and probability of a mistake of threshold body (with the indication of intervals of values of these sizes and probabilities of their hit in these intervals).

In machine experiment the distributions of probabilities of a mistake of threshold body were received during adaptation on algorithm UEIP with a feedback at

$$n = 8, q_1 = 10^{-3}, q_2 = 10^{-1}, q_3 = q_4 = 5 \cdot 10^{-1}, q_5 = q_6 = q_7 = q_8 = 8 \cdot 10^{-1}$$

and $q_9 = 5 \cdot 10^{-1}$, when exact value of probability of a mistake of restoring system for optimum values of weights and a threshold makes $Q_{min} = 7{,}912 \cdot 10^{-4}$.

In all three cases the initial weights and a threshold were set zero, size β made 0.05, value of the activator of the generator of random numbers was 33, and the amount of steps of adaptation- 100, 500 and 500, accordingly.

From the results received by us it follows, that (at correct selection of initial weights, a threshold and sizes β of their increments with growth of the amount of steps of adaptation maxima probability distributions of conditions to which various probabilities of mistakes of threshold body answer, are shifted to the minimal value in the given conditions Q_{min}.

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